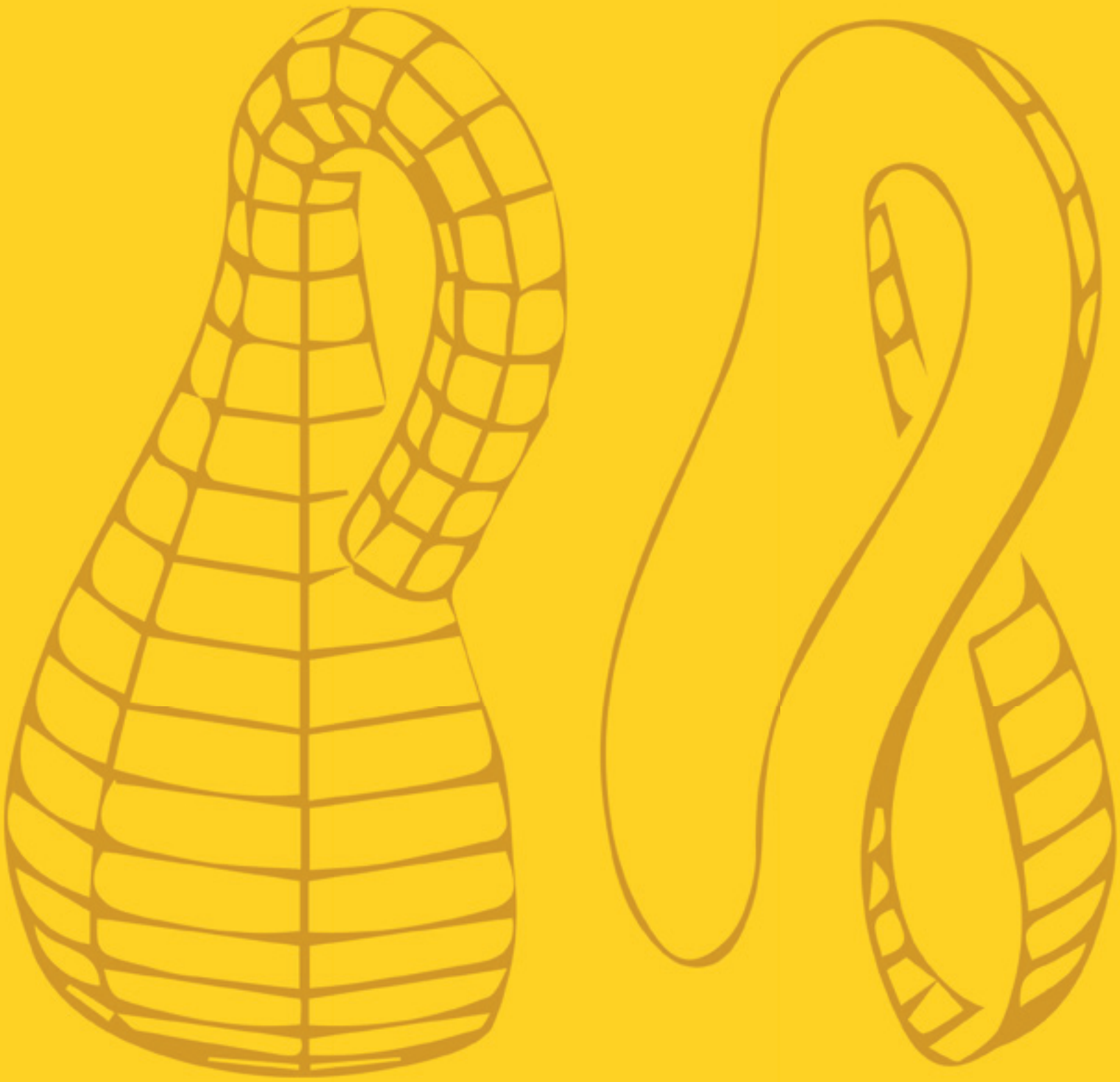


OUTSIDE IN

OUTSIDE IN

INSIDE OUT

FELICITAS ROHDEN

TURNING A SPHERE INSIDE OUT

An Artistic Perspective On Sphere Eversions

Felicitas Rohden

In 2021 I realized an installation and artist book entitled “Where the Sun Sets Blue.“ It was released in tandem with the interactive website http://www.wherethesunsetsblue.com.
The project was based on a year-long correspondence between myself and scientist, Sukjin Han, who was the commander of NASA’s Hi-SeAS Mars Mission VI in 2018. In an attempt to send the first humans to Mars, scientists are running simulation programs on volcanos and in deserts on Earth to understand how living on Mars would look like.

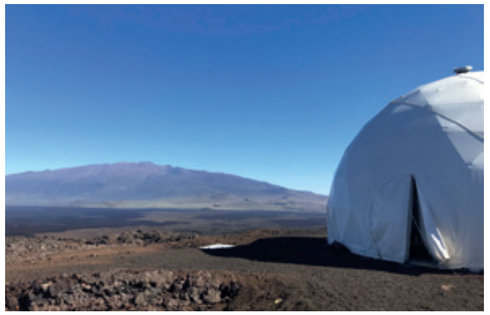


Fig. 1

The HI-SEAS (Hawai’i Space Exploration Analog & Simulation) is a long-term experiment sponsored by NASA and operated by University of Hawai’i, starting from 2013. Each mission is 8 to 12-months of extreme isolation where a group of scientists live together in a simulated Mars habitat located on the Mauna Loa volcano. The purpose of the simulation is to examine what physical and psychological challenges a crew faces in the extreme isolation.

Fig. 2

Our conversations resulted in 6 round books, each written and designed from the perspective of the five senses: Taste, Smell, Hearing, Touch, Sight and Synesthesia. It is an attempt to understand the material grammar of a new world.

Fig. 3

Similar to the habitat, the book is small and secluded. As you read, you are forced to rotate the book evoking orientation loss much like the scientist may have felt during his mission.
In addition to discussing the purely tangible experiences of Sukjin, it was important for us to abstractly grasp the concepts we had gone in depth about. In doing so, it was an easy decision to construct the books much like how planet Mars is constructed. The reader opens up the book and delves deep into not only the psychological nature of the people involved in the mission, but also into the planet itself, albeit from a subjective and abstracted perspective. The reader is transported and symbolically present on Mars and is given the awareness to experience it from a unique standpoint.

The immersive web project ‘wherethesunsetsblue.com‘ also translates this experience into the digital sphere.
Starting from an digitally sculpted Martian surface, the website makes it possible to interact with the spheres thus defined by sensory perception. Visitors can enter into the spheres’ interiors, read the dialogues, and see images from the Mars habitat.
When hovering over the spheres, the generic mesh appears and thematizes the digital immateriality of the objects.

This artist book entitled “Inside Out - Outside In“ is a prolongation of this project and explores a possible translation of the 6 digital books when they are converted back into actual space.

I was interested in further investigating the nature of a digital surface; in my case the tension field between the generic mesh and the martian soil. It is a surface which appears to be something else than it is. I like to tackle questions of dimensionality: What is surface? What is substance? What is a grid? What is volume? Where do the two coincide?

To bring the digital surfaces back into an actual space, I applied a mathematical method called “Sphere Eversion,“ which describes the process of turning a sphere inside out. It is a continuous deformation, which allows the surface to pass through itself, but forbids puncturing, ripping, creasing, or pinching the surface. There are several ways of producing explicit examples and mathematical visualization called halfway-models. This is an immersed spherical surface which is halfway inside-out, in the sense that it has a symmetry interchanging the two sides of the surface.

These halfway-models have been an inspiration for turning the 6 spherical books inside out, as well as the design of this artist book which has 2 covers and 2 sides. The first part focuses on the outside view, whereas the second part shows the inside of the shapes. The wirebinding enables the reader to flip the book in 2 directions.
This artist book gives a perspective on the simulatiousness of inside and outside, mesh and soil.
Both sides of the book start with a view of the immersive website and the location of the digital spheres in outer space. In a next step, the sphere is being inverted and shows the changes of its surface and volume. Mathematical surfaces such as Genus 1, Klein Bottles, Morin and Boy‘s Surface appear. These new shapes are also a reference point for a series of room filling inflatables called “Inverted Land.“

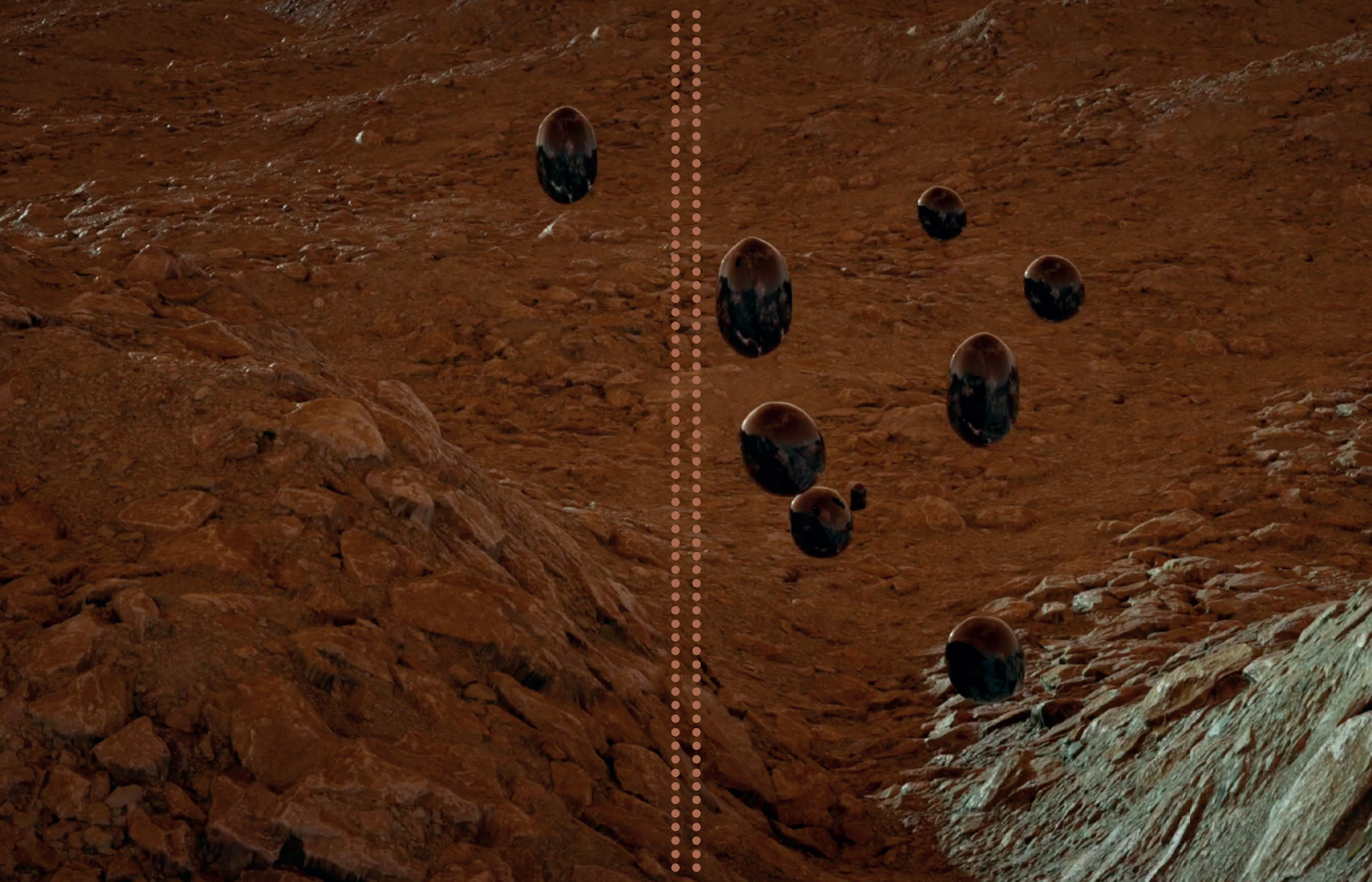
Fig. 4

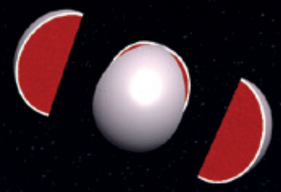
The conception and realization of a project like this depend on the support and commitment of several people whose professionalism, collegiality and generosity I have enjoyed as an immense privilege. I would like to thank Kim West who helped me realize the 3D renderings and spend hours helping me visualize my ideas.
Special gratitude goes to mathematician Rudi Penne who wrote a fantastic text for this book. Our ongoing conversations have sparked my imagination in the past years and helped me understand a certain mathematical logic.
I am especially grateful to Sukjin Han, Luke Calzonetti, Angelika Trojnarski, Meike Denker and my family.
This book could not have been produced without the generous support of BBK (Beauftragte der Bundesregierung für Kultur und Medien) and Neustart Kultur.

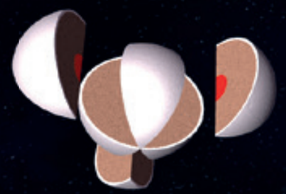
Image Credits:

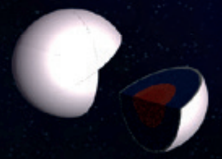
Fig. 1 HI-SEAS Mars Habitat, Photo: Sukjin Han
Fig. 2 Installation view “Where the Sun Sets Blue“ Photo: Bernhard Adams
Fig. 3 Installation view “Where the Sun Sets Blue“ Photo: Felicitas Rohden
Fig. 4 Installation view “Inverted Land (Hearing)“ Photo: Kai Werner Schmidt

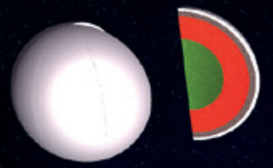
www.wherethesunsetsblue.com

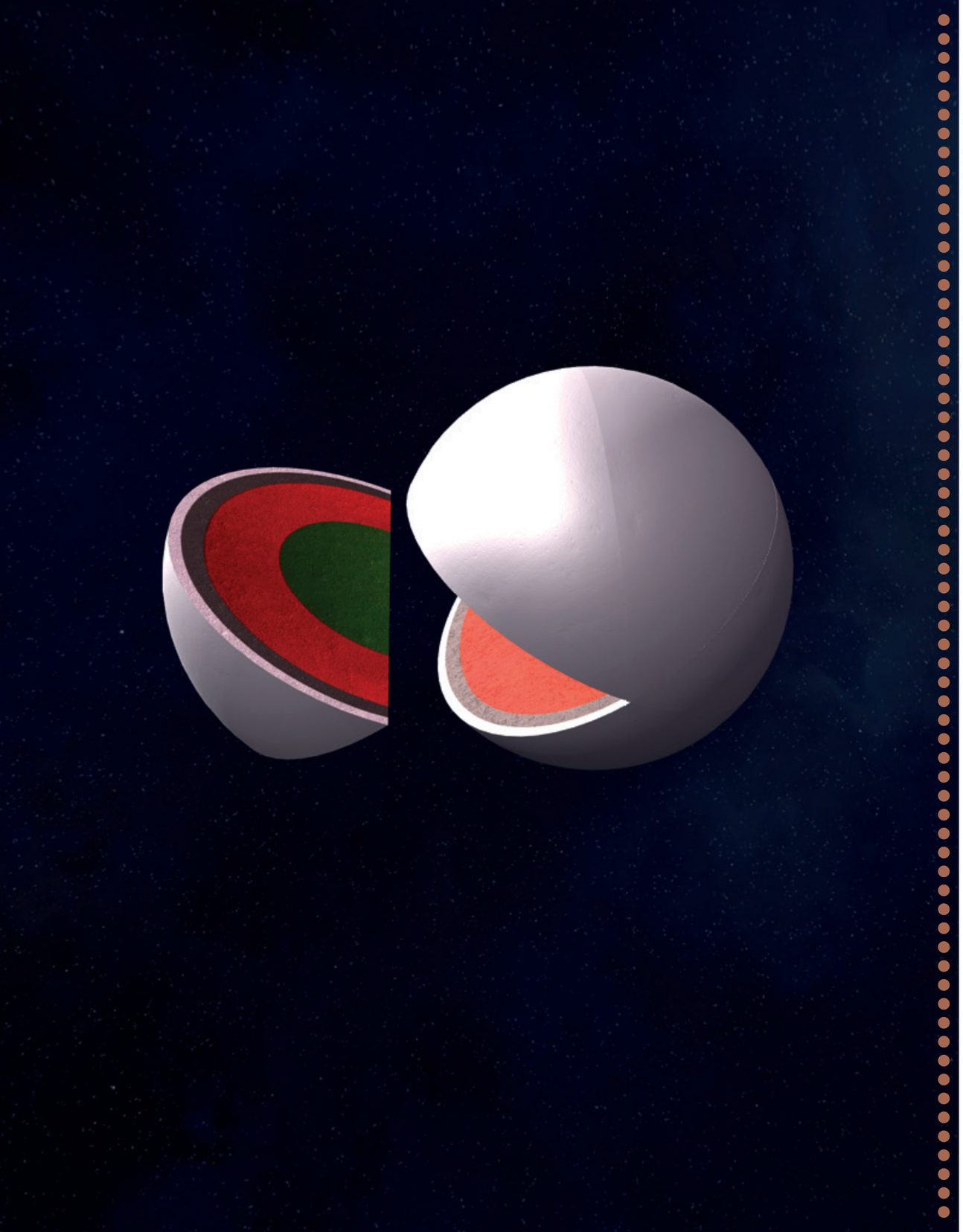












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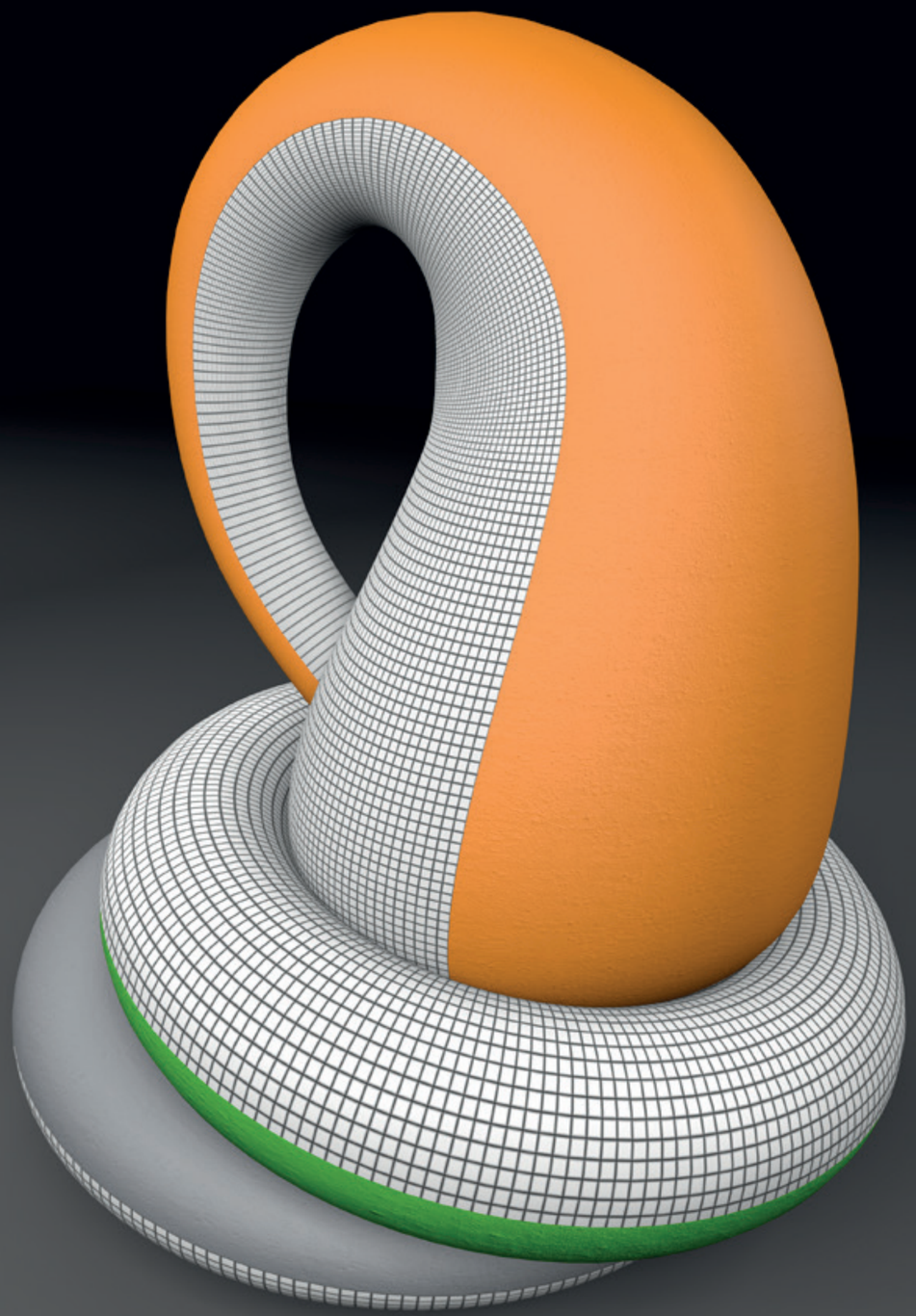

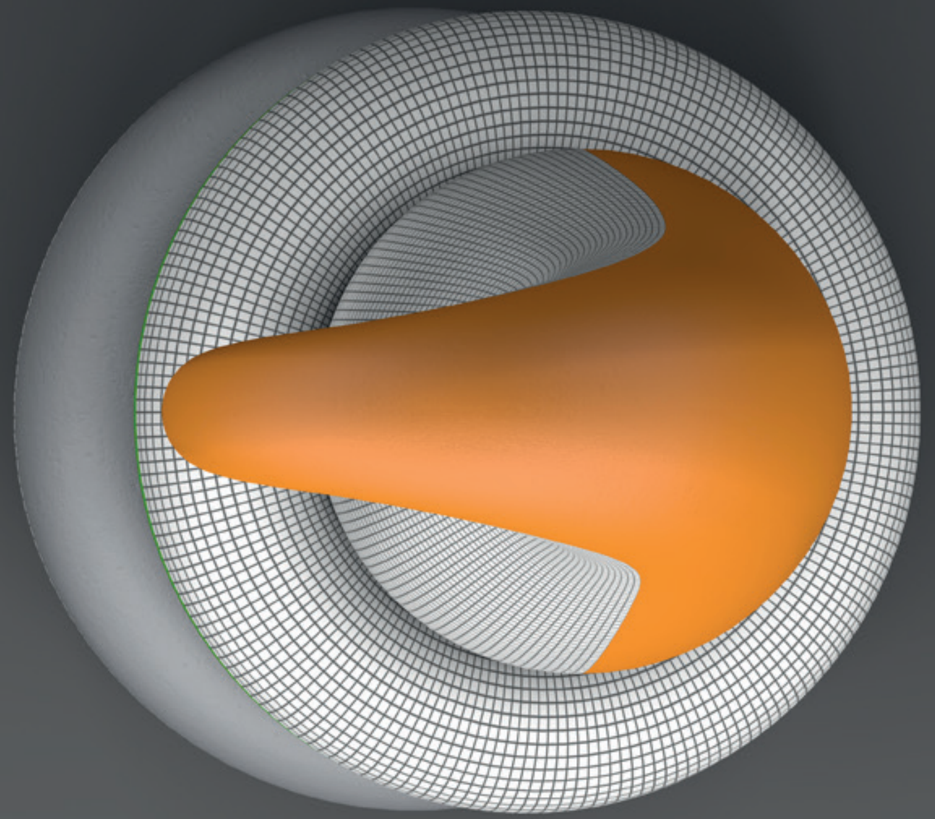

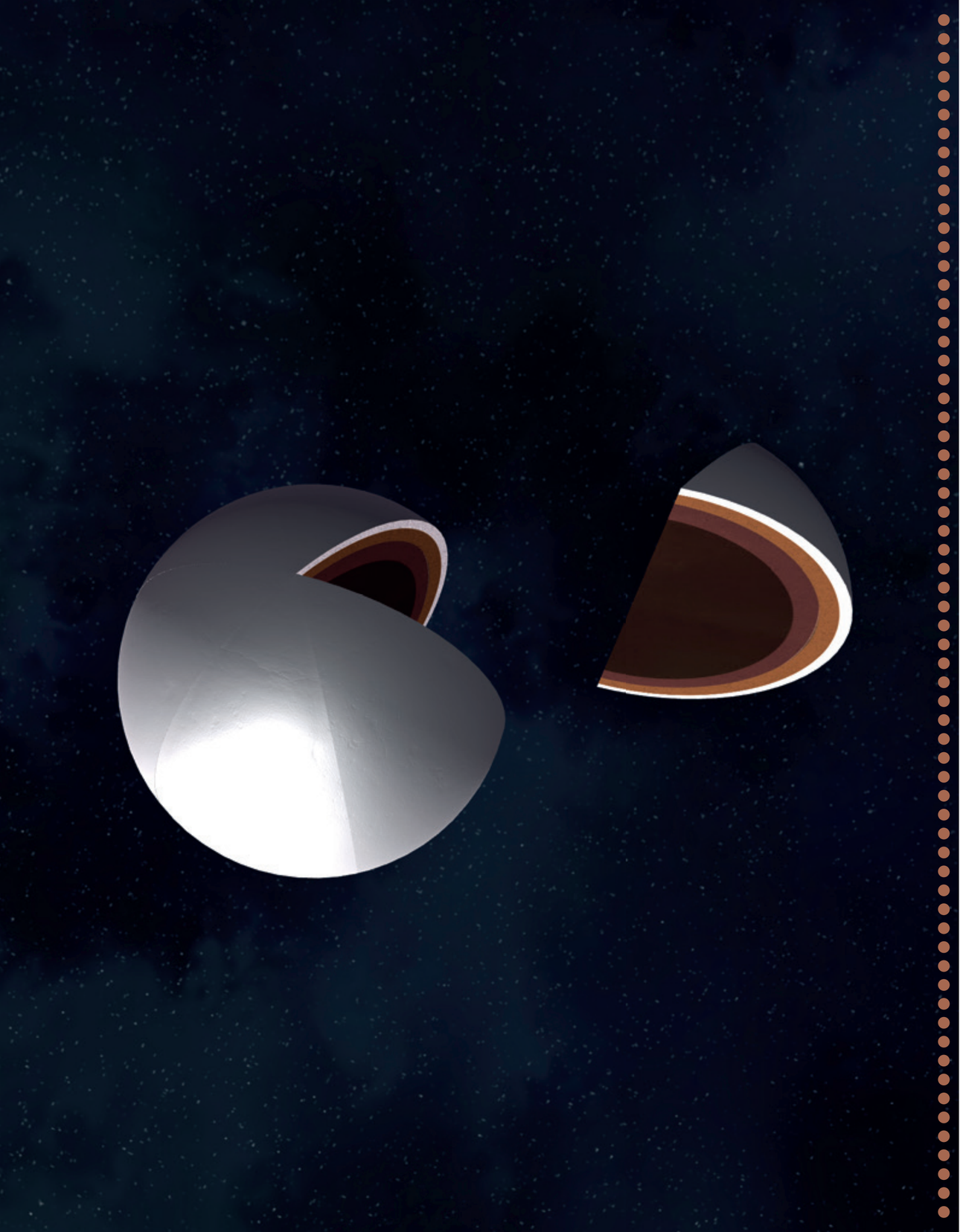

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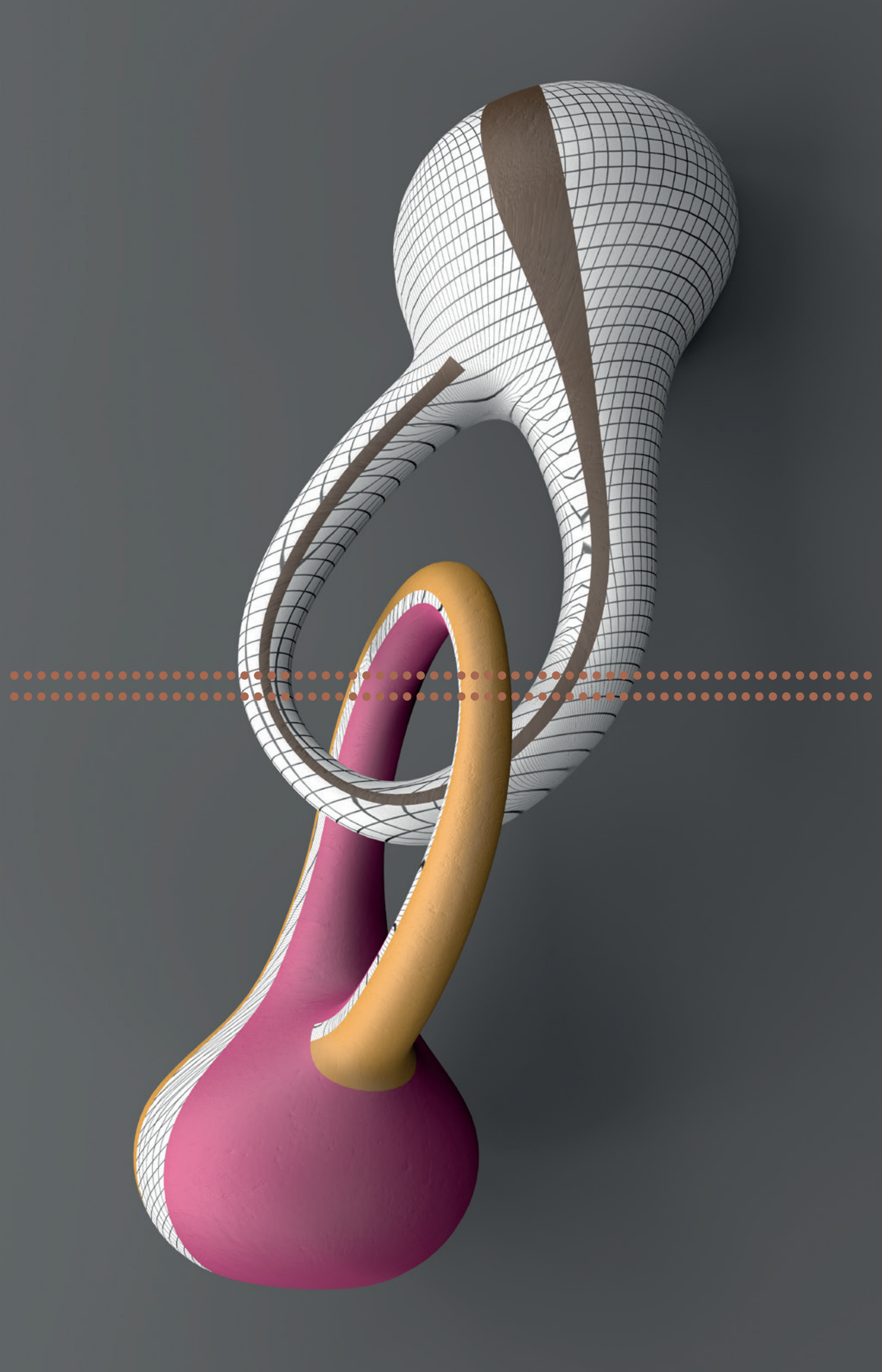

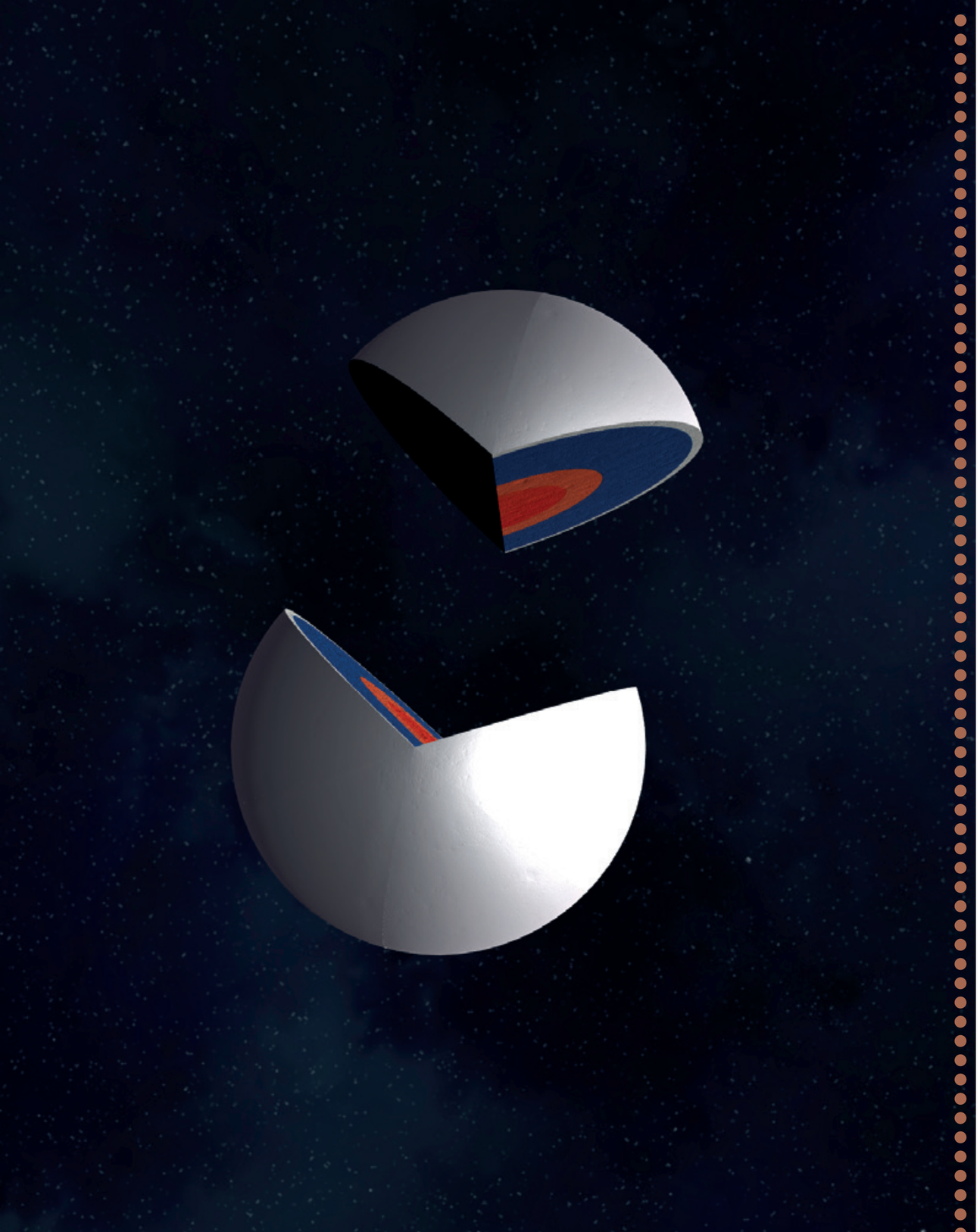

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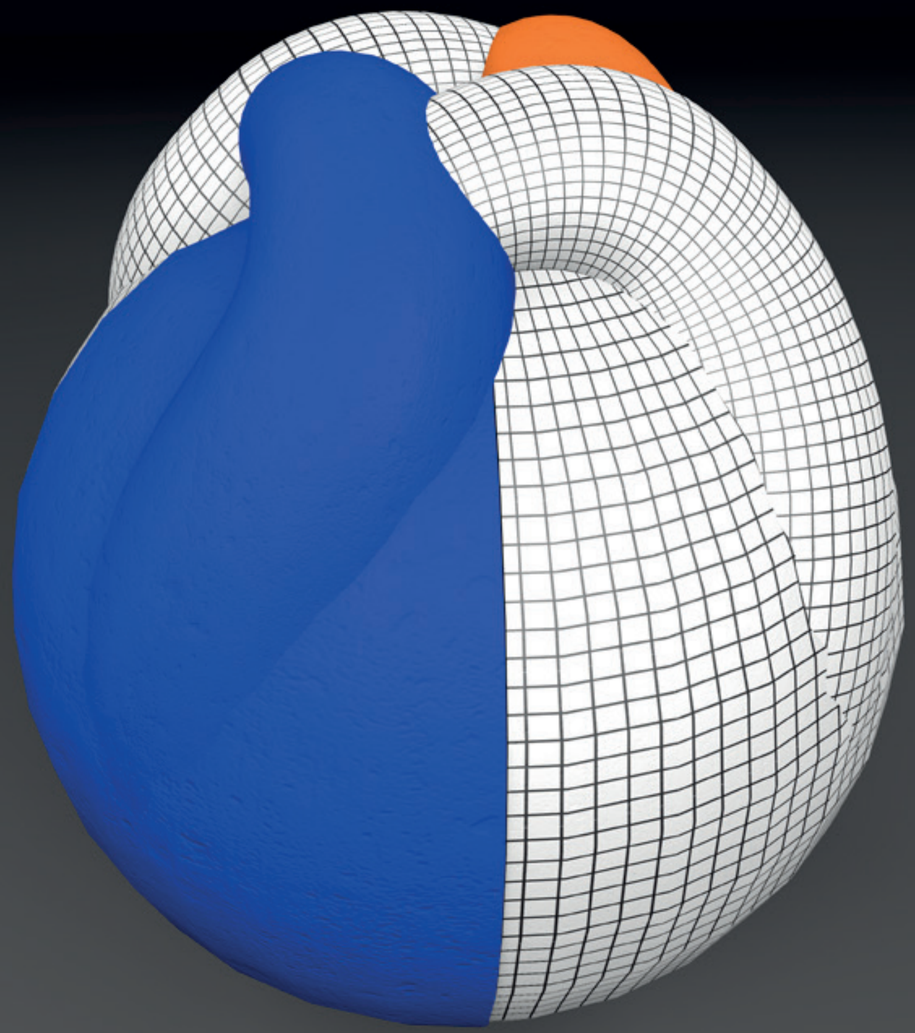

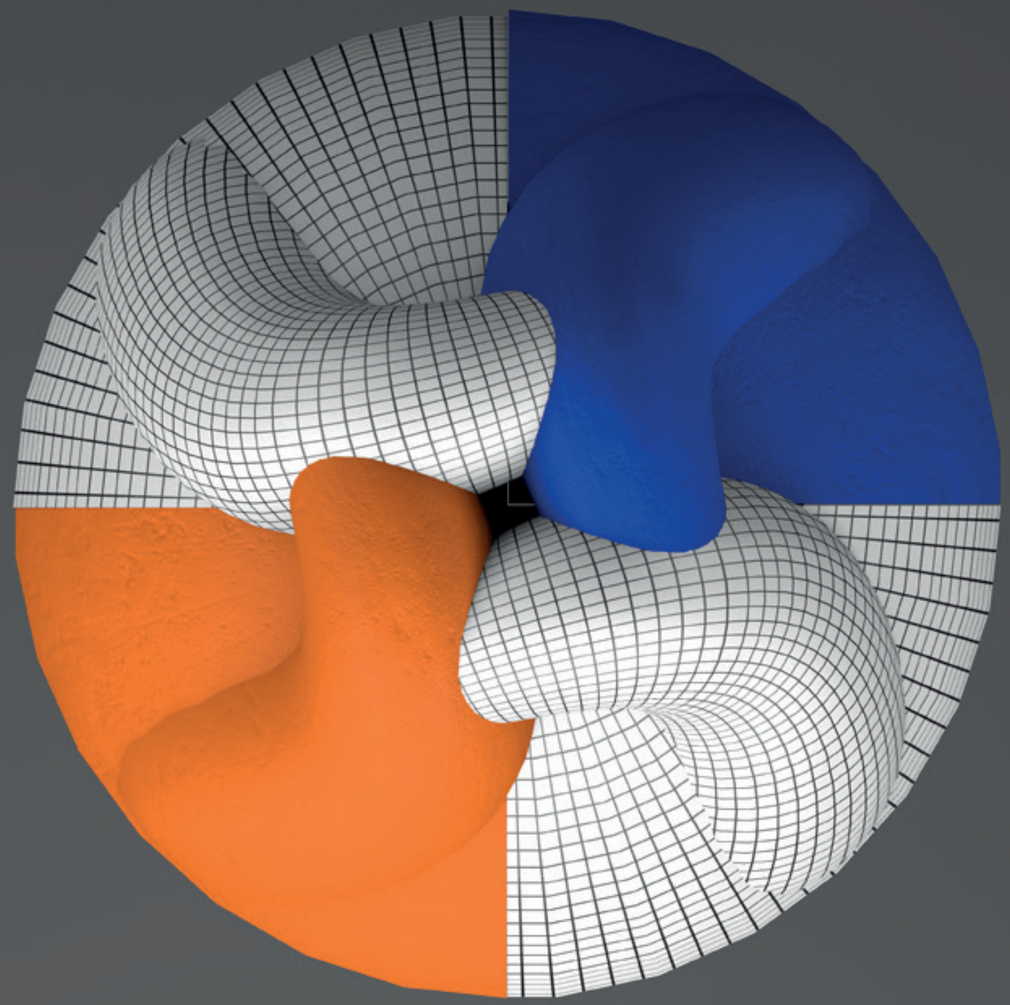

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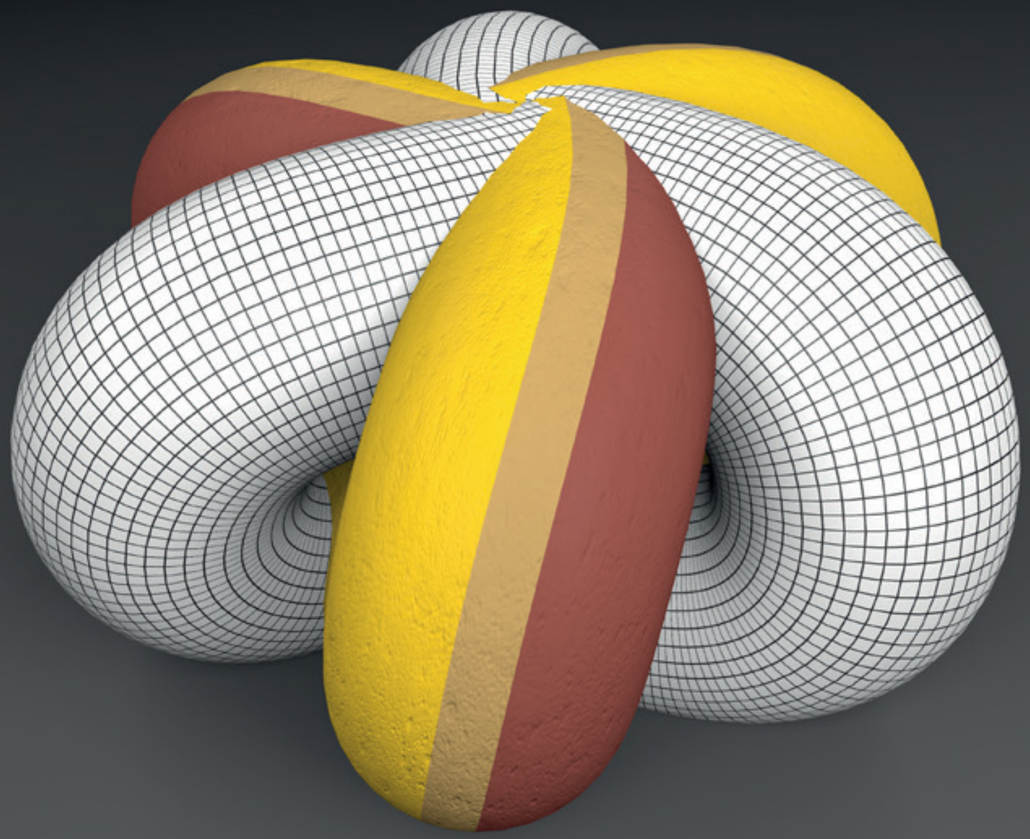

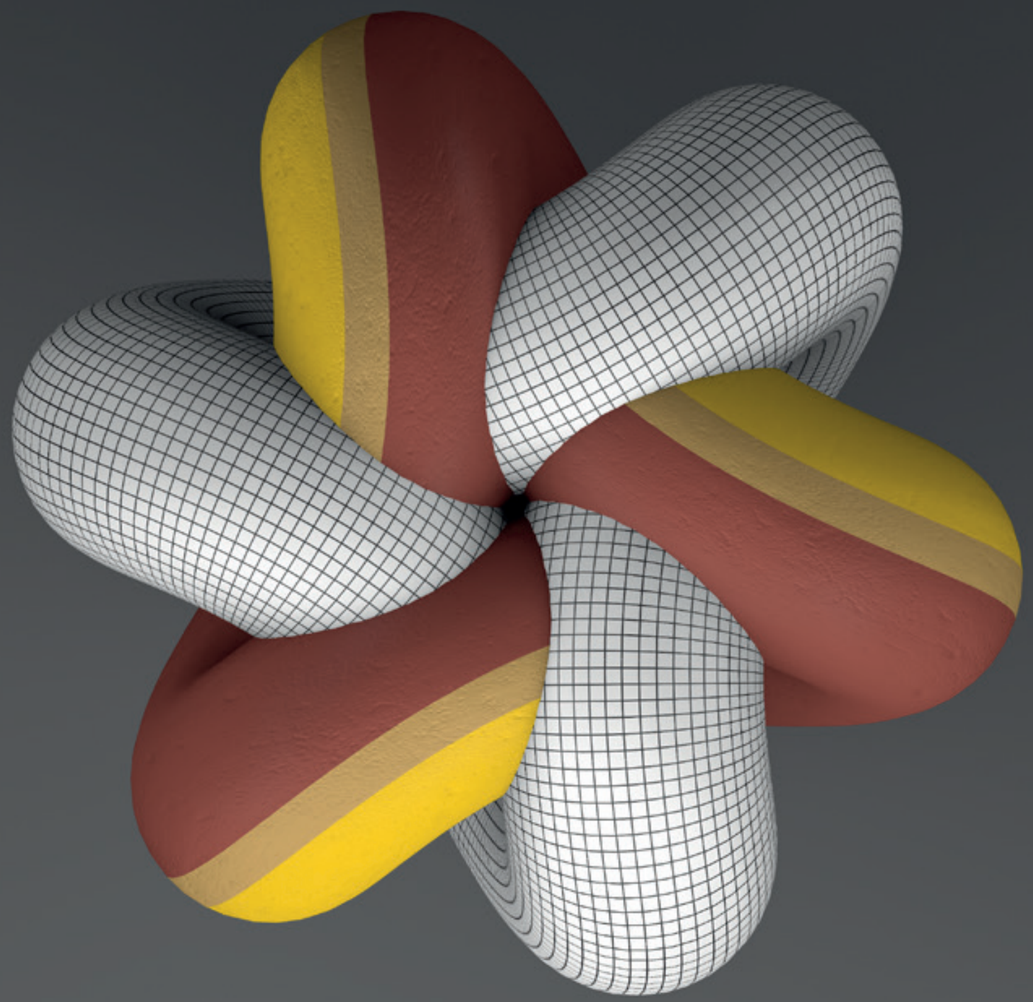

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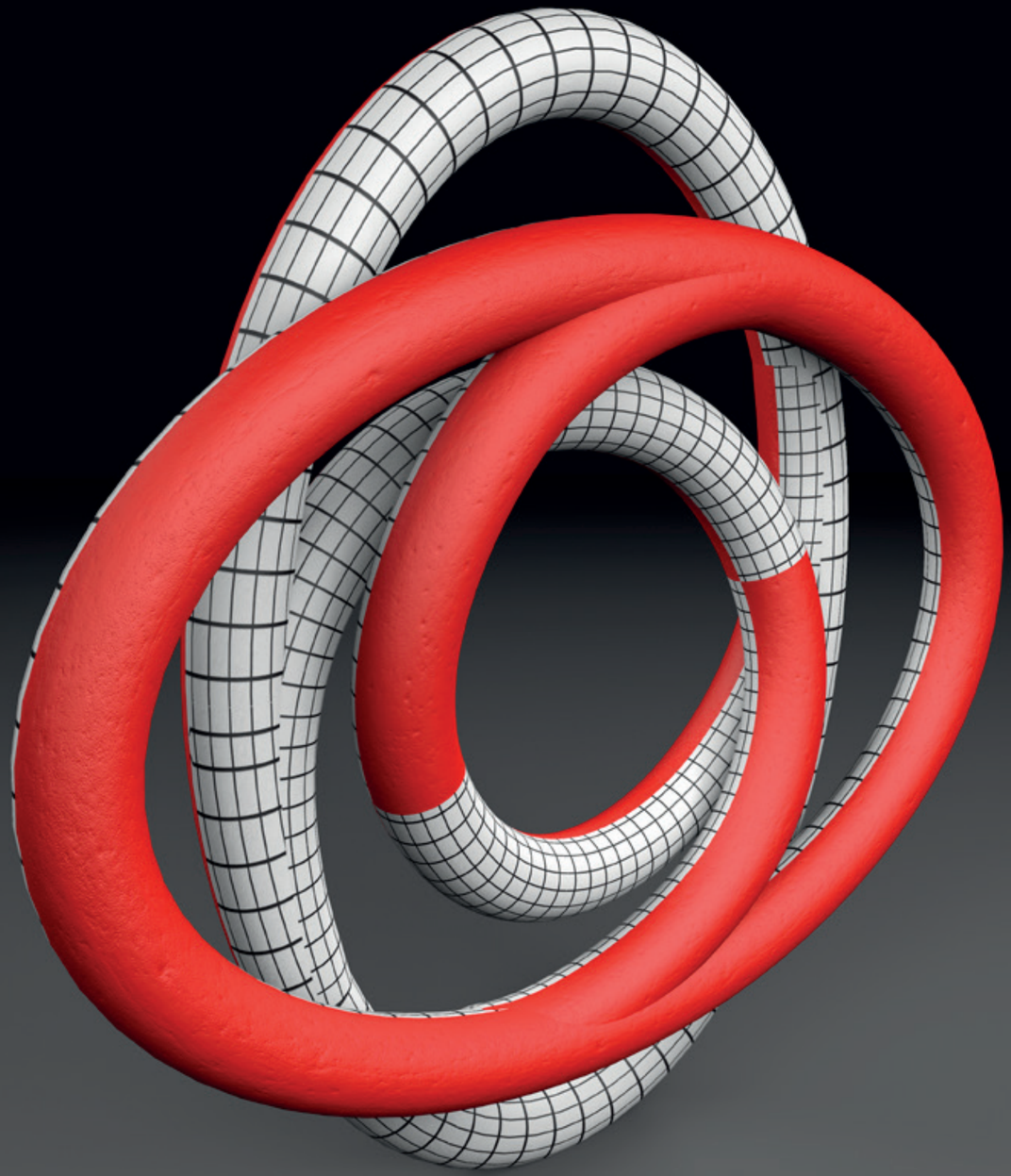

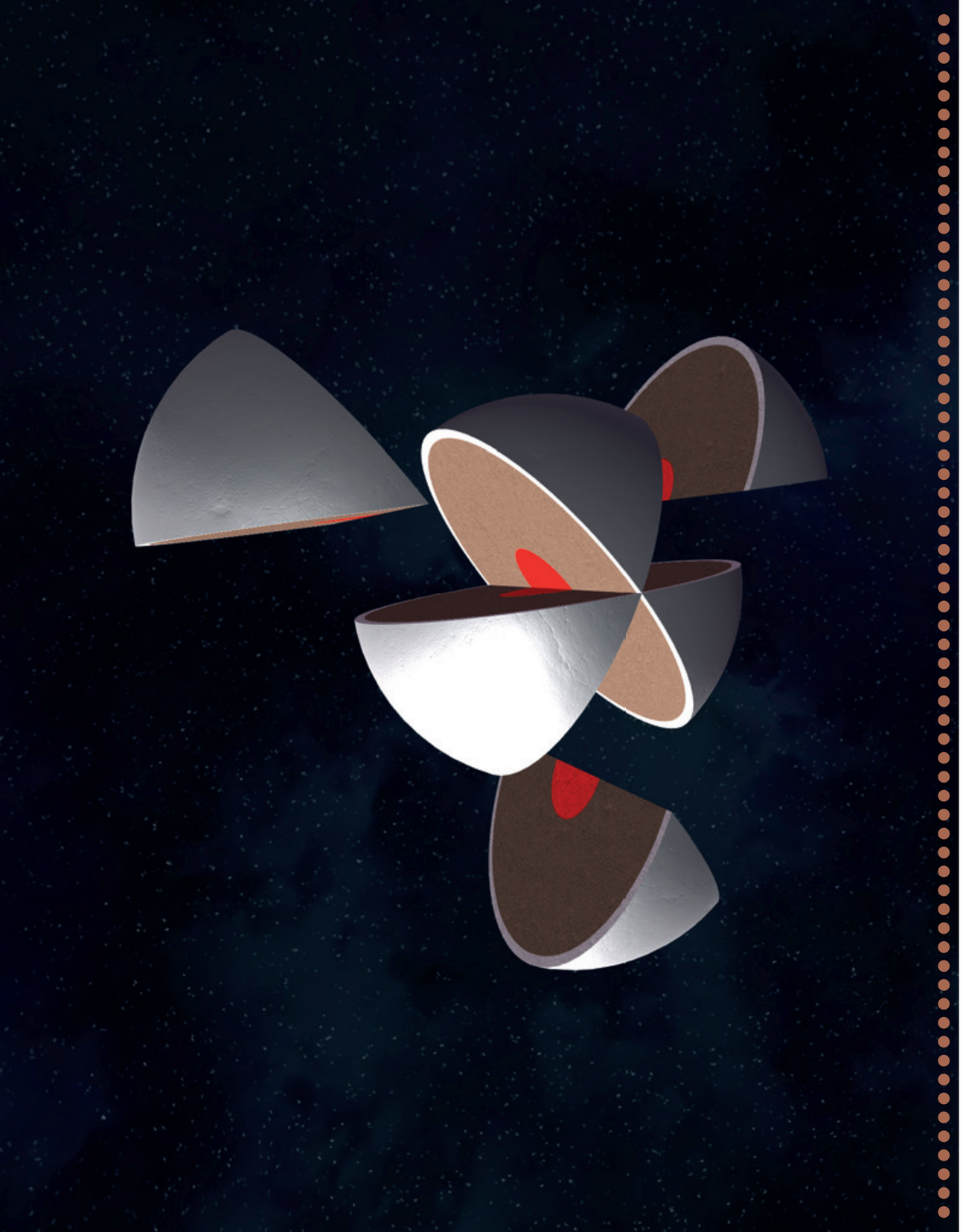

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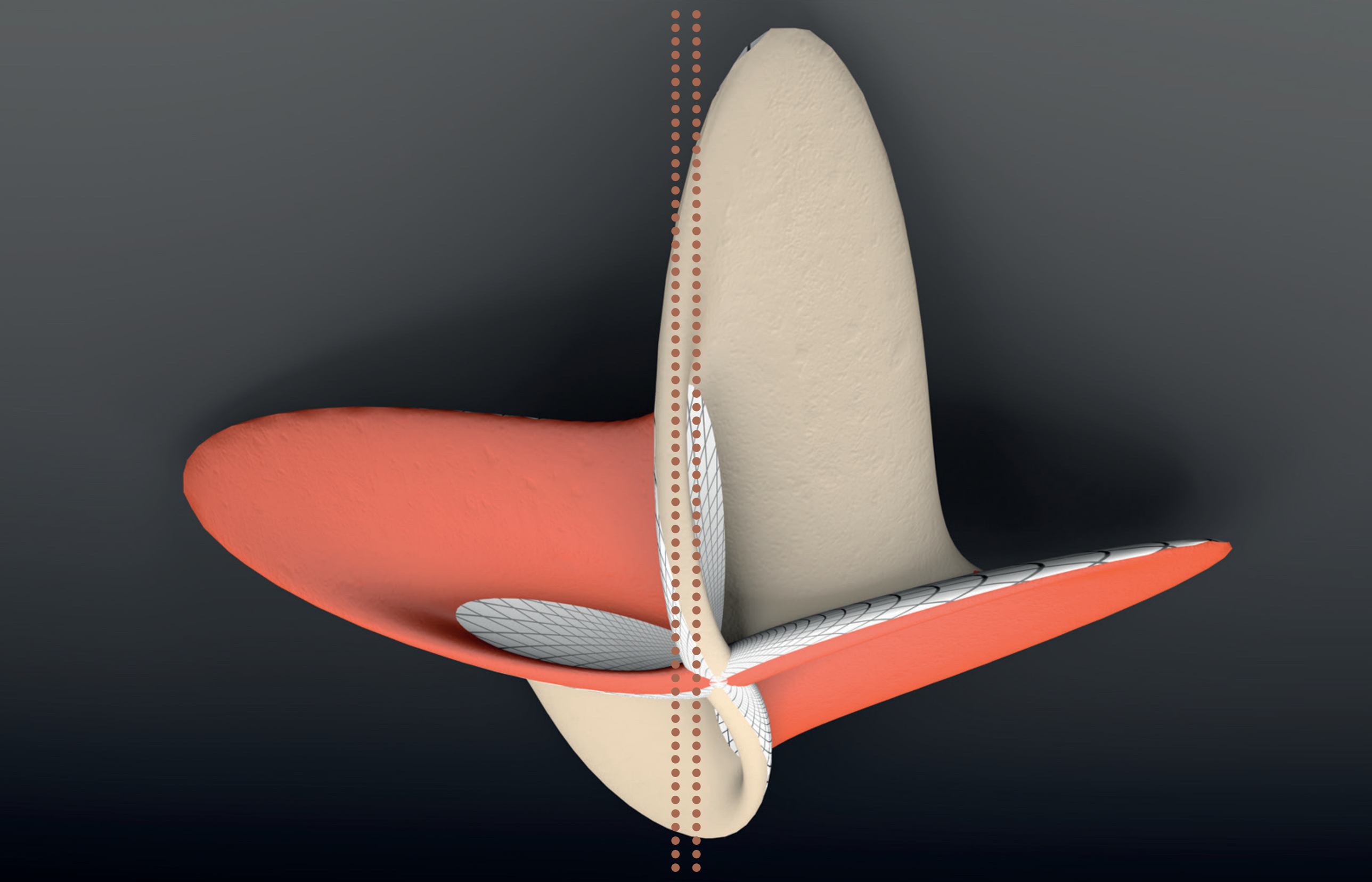
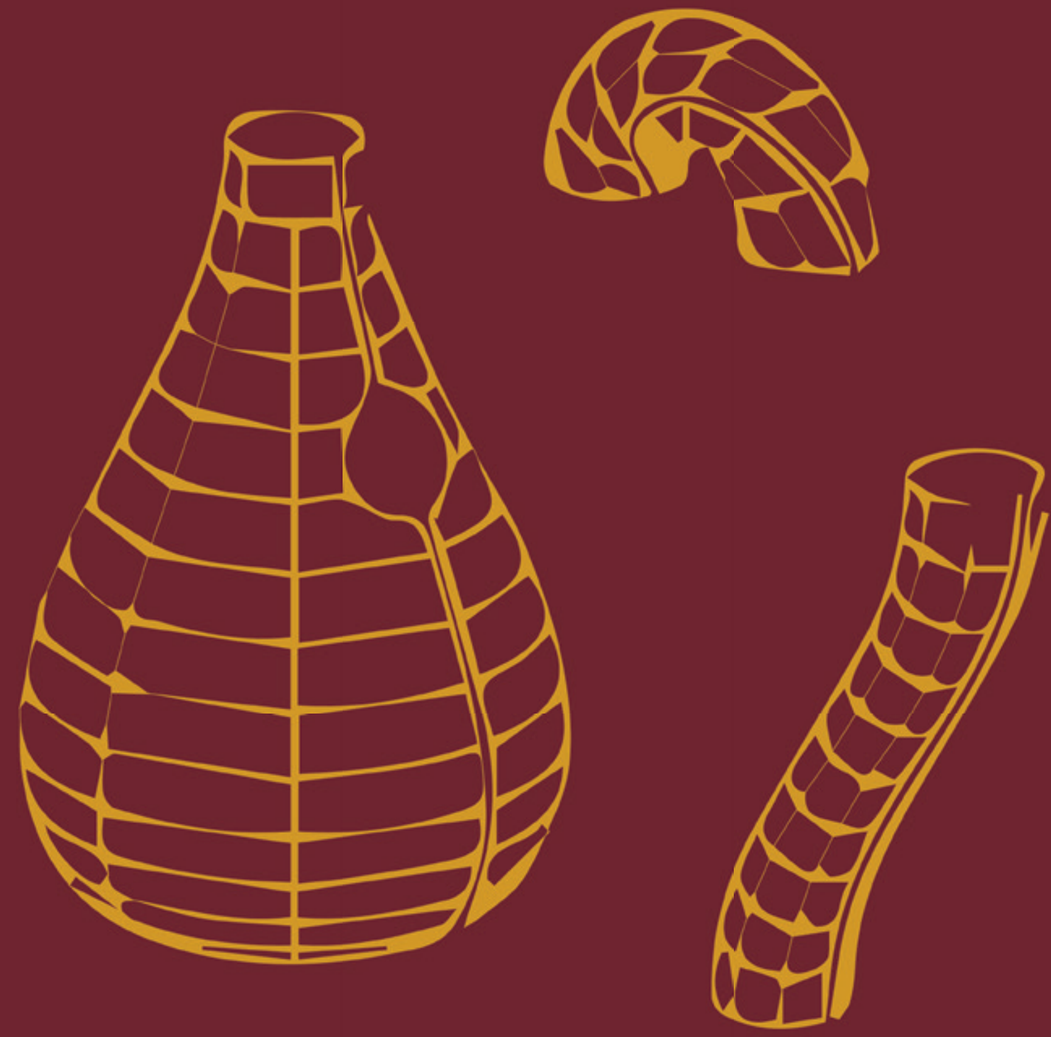


This book has 2 covers - “Outside In“ and “Inside Out.“ Flip to the cover you prefer and use it as your starting point.



INSIDE OUT

WHAT IS THE IDENTITY OF AN OBJECT?

A Mathematical Perspective on Invariants of Transformations

Rudi Penne

Sometimes we consider two objects as identical, sometimes we decide to distinguish between them. This decision is motivated by the (often unconscious) choice of transformations and deformations we are willing to allow for an object while still considering it to be the same, that is, without losing its essential characteristics (identity).

In our daily life, the choice of feasible transformation depends on the context or on the observed subject. But mathematicians and artists aim to transcend common contexts and perceptions. They do not limit the possible transformations and deformations and use experimental choices to redefine the identity of an object. In mathematical terminology, this challenge is called the theory of invariants for transformation groups. Recognizing these invariants is a central theme in the art of abstract thinking.

For example, in Euclidean geometry a circle is considered to be a different concept as an ellipse. But from the point of view of a photographer or perspective graphic artists, circles can appear as ellipses. The mathematical framework for this other reality is given by Projective Geometry.

Fig. 1

Living in an elastic world: Topological Invariants

Another illustration is given by polyhedral objects, bounded by edge-sharing polygons. See for example the figure below showing the 5 Platonic polytopes.

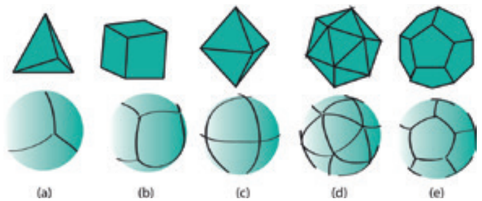


Fig. 2

The context where these objects pop up dictates which properties are considered to be essential.
For example, when polytope (b) appears in design or construction applications, we often expect a real cube with 6 identical squares and with 3 mutually perpendicular edges meeting in each of the 8 vertices. But equally often, the condition that edges are equal and/or orthogonal is less important, as in the perspective context of Projective Geometry. It might even be the case that only the combinatorial structure of the edge framework matters. The essence of a cube is then reduced to a network of 8 nodes, each of them connected to 3 neighbors using 12 links in total. The combinatorial structure of a dodecahedron (e) is given by 20 nodes, each having 3 neighbors, with a total of 30 links (edges).

Going one step further in the search of the essence of an object, we neglect the geometric measurements and the combinatorial vertex-edge structure, maintaining the fundamental shape of the object. This is the point of view in Topology, a mathematical area where objects are considered to be made of flexible materials such that they can be inflated or arbitrarily deformed, as long as we do not use knives, scissors or glue.
The artistic works of Felicitas Rohden apply this topological perspective and question what would happen if you apply this mathematical principle to materialized objects.

In this topological context, the platonic (and other) polytopes are considered to be equivalent, all having the same spherical shape. On the other hand, the sphere and the torus (donut) are recognized as different topological objects, because the “hole” in the torus can never be realized by any elastic deformation of the sphere.

Fig. 3

Actually, for “closed surfaces in 3D” (no borders) the number of holes (or handles) is an invariant under topological deformations, called the genus of the object, and can be considered as the DNA of the shape.
Along these lines, the frog below will be identified as sphere-like (genus 0) and the cup as torus-like (genus 1):

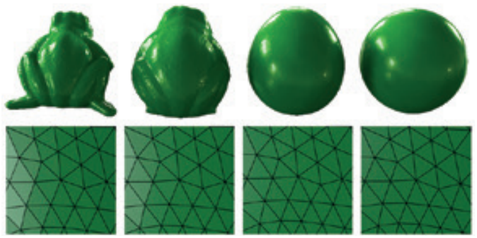


Fig. 4

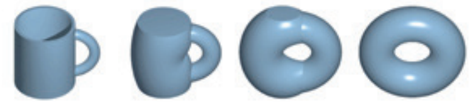


Fig. 5

It is interesting that in many applications (computer animations, manufacturing, 3D-printing…) we still make use of geometric structures (with vertices, edges and faces) to represent a shape. Popular polyhedral presentations are grids or triangulations, but these are random supporting tools and have no impact on the shape-DNA:

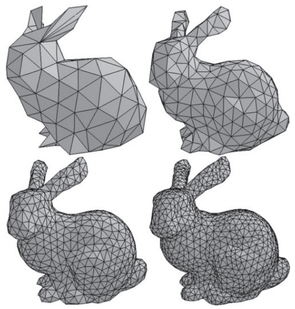


Fig. 6

A nice mathematical discovery is the formula that computes the genus of an object by means of a simple count (the Euler characteristic): $\chi = v - e + f$, where v is the number of vertices, e is the number of edges and f is the number of faces. More precisely: the genus G is determined by

$$G = (2 - \chi)/2$$

This implies that the Euler characteristic of an object does not depend on the triangulation or grid resolution, because its value is hidden in the shape-DNA. It is a topological invariant. Check yourself, the result of $\chi = v - e + f$ for each of the Platonic bodies will be 2, which corresponds to genus 0 (spherical). This was already observed in 1758 by the mathematical wizard Leonhard Euler. Likewise, a triangulated torus always yields $\chi = 0$.

Orientability and Turning Inside Out

There is another interesting topological invariant, called orientability. This is most easily explained with a triangulated model of a closed surface. Such a surface is called orientable if we can attribute an orientation to each triangle (= fixing a traversal sense of the boundary) that is “compatible” with the orientation of its neighboring triangles, in the sense that the induced walkthrough for an edge is **different** with respect to both adjacent triangles.

Fig. 7

If the property of orientability holds for some triangulated model of the surface, then this is true for any other triangulation, because it is an inherent topologic characteristic.
Orientability of a surface is closely related to the inside-outside concept. Indeed, if we image a closed surface as the boundary of a 3D-object, then we can agree to orient the triangles counterclockwise when viewed from the outside, meaning clockwise for an inside observer. This strategy always yields compatible oriented triangles.
In 1882 the German mathematician Felix Klein offered the world the famous Klein bottle.
You can see several variations of Klein Bottles in Felicitas Rohden’s inflatables, objects and 3D animations, which can be seen in this book.

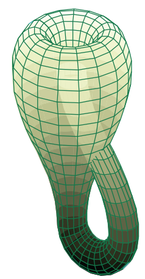


Fig. 8

This too is a closed surface, having 2-dimensional grid or triangle representations. But it appears that we need 4 dimensions to realize it. If we try to build a Klein bottle in 3D, we get stuck by space limitations and have to cheat by allowing self-intersections. This means that certain different points of the Klein bottle are drawn as one point in 3D, which is no correct representation. Nevertheless, we mostly sin in illustrating the Klein bottle, because 3D is more familiar to us than 4D. Alternatively, we can model the Klein bottle by a triangulation, listing all participating triangles, mentioning which triangles share edges or vertices. Observe that for the Klein bottle we cannot distinguish inside from outside. The Klein bottle does not bound a 3D-object. Another way to say this: the Klein bottle is not orientable, which can be checked by the triangulation model.
A curious fact is that the Euler characteristic of the Klein model (computable by a given triangulation) equals 0, which means that it shares some DNA with the torus (also Euler characteristic 0). But since the torus is orientable and the Klein bottle is not, we classify them as different shapes.

In this article we showed the impact of all kinds of transformations on objects (scaling, changing perspective, …), searching for invariants in order to determine the essence (DNA) of the objects, under varying context conditions. We even considered topological characteristics that withstand the flexible deformations of rubber-sheet geometry. Artists and mathematicians share the tradition to push and cross the existing boundaries. What if the bounding surfaces of objects could be moved such that they are able to penetrate themselves? In the illustration below, the purple inside of a sphere has been pushed outside the sphere, without destroying anything. Notice that orientability is not maintained anymore, because we lose the concept of inside and outside.

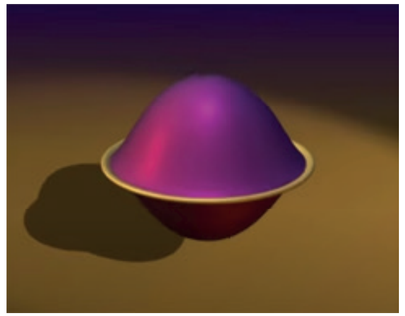


Fig. 9

During these smooth (sometimes self-penetrating) deformations we never cut anything or attach new parts, otherwise each object can be obtained from any other object. Furthermore, we avoid creating creases during the transformation, because they prevent us from pulling further (this would make material disappear).

So, it seems that in spite of the extra transformation freedom (self-intersecting), the no-crease rule prevents us from turning the sphere inside out. Nevertheless, in 1957, Stephen Smale proved that this so-called sphere eversion was possible. This result is contra-intuitive, and in the beginning many mathematicians believed it was wrong. But in the meantime, several convincing visualizations are available of this spectacular inside out transformation, showing curious intermediate surfaces that are not orientable.

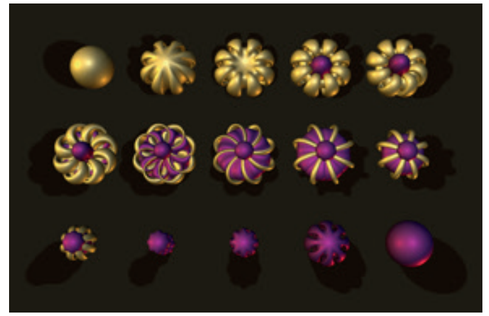


Fig. 10

The 6 sphere eversions that can be seen in this book, depict several intermediate surfaces known as Boy’s surface, Morin Surface, Klein Bottle and Genus 1 surface.

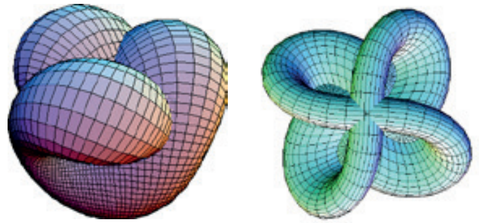
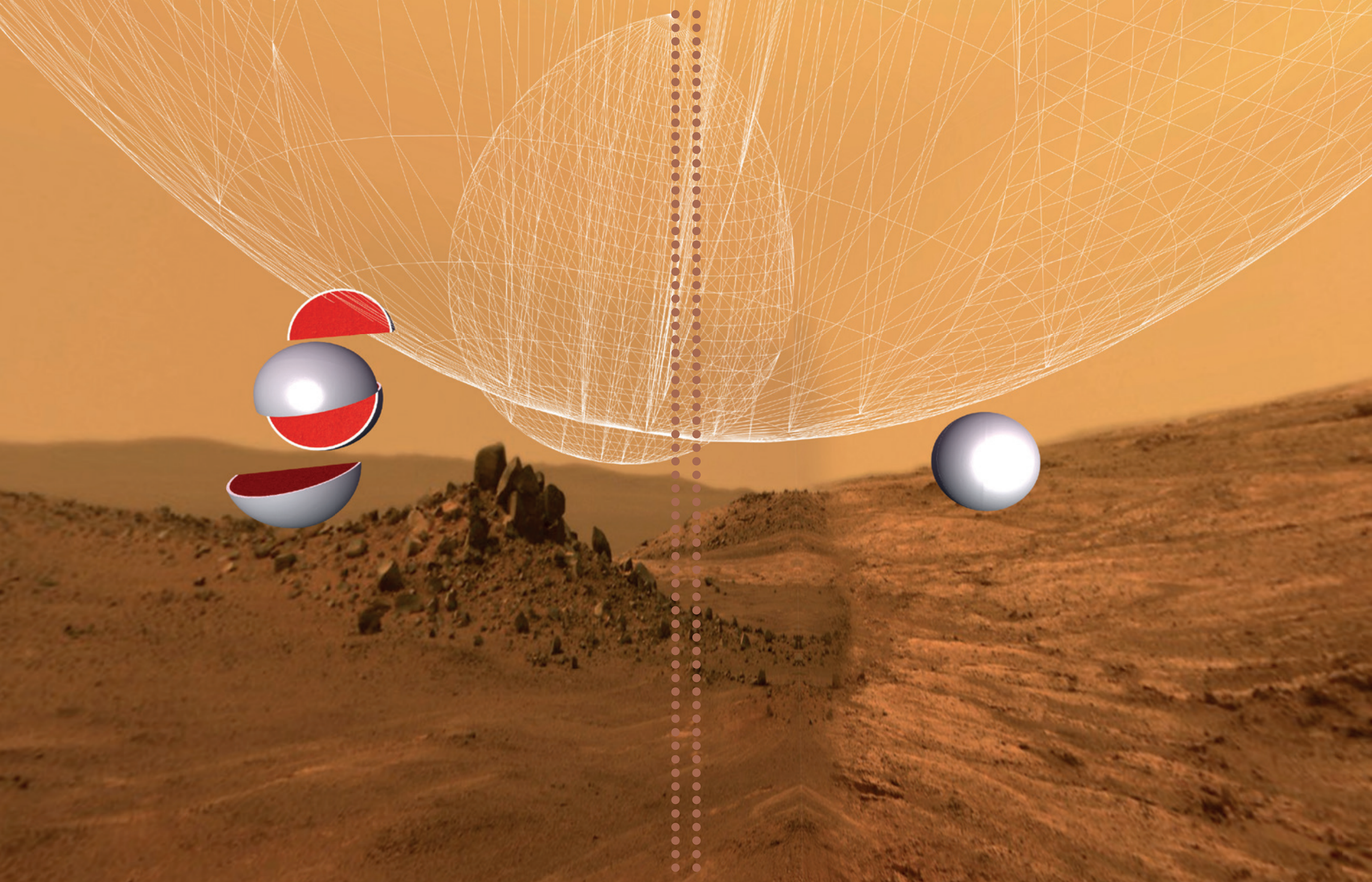


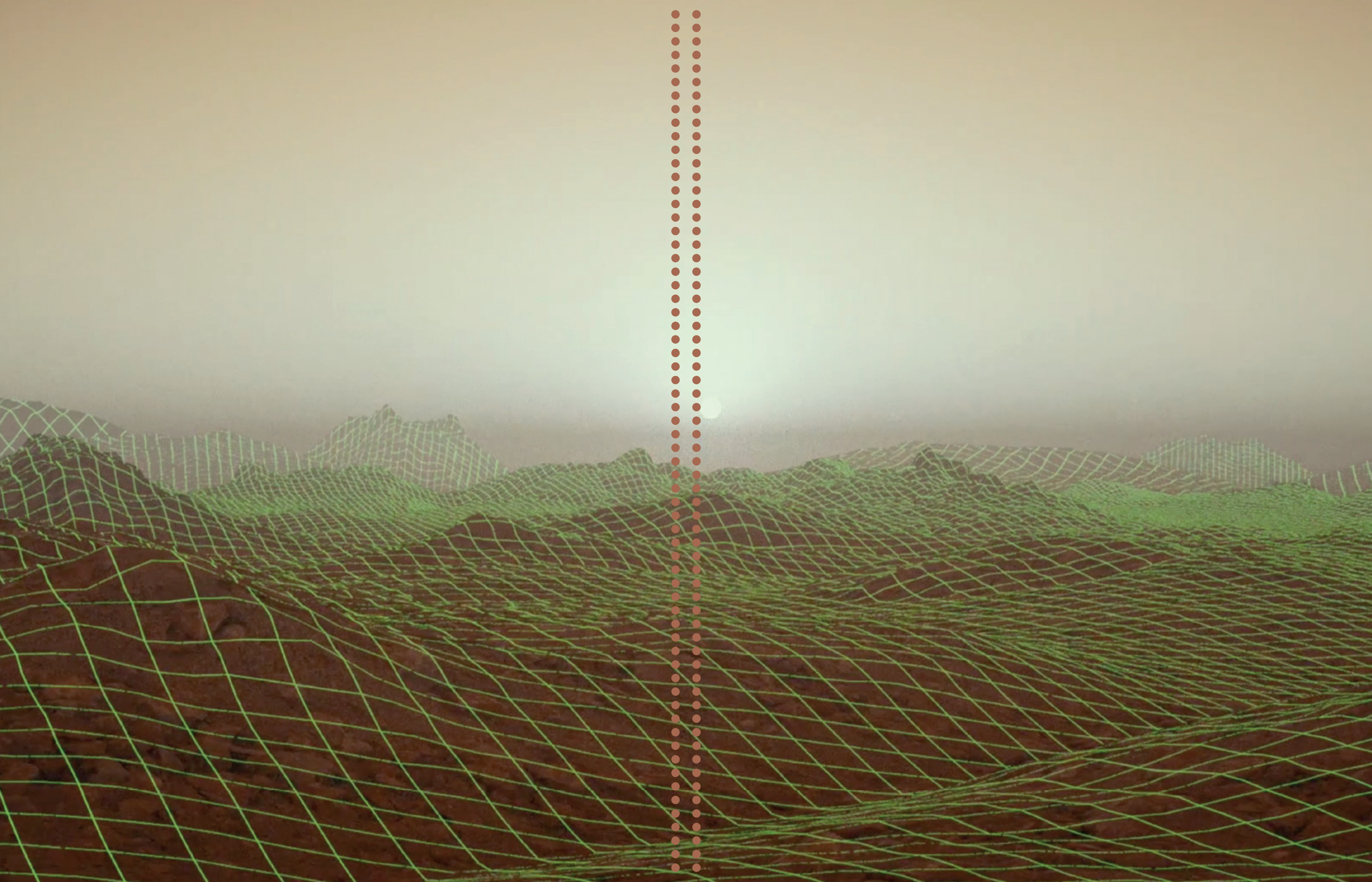
Fig. 11&12

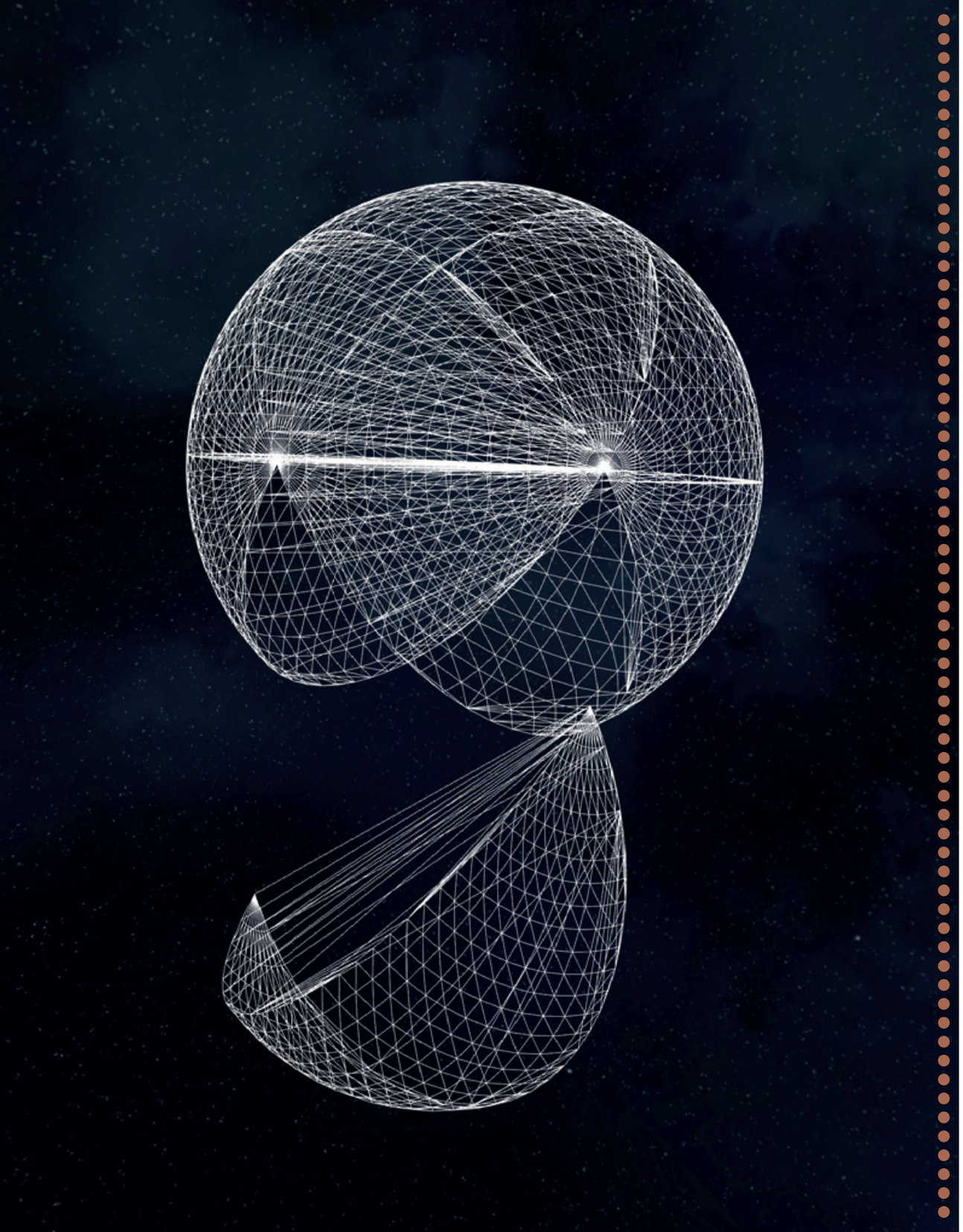
These shapes represent the simultaneity of the inside and the outside. In the examples of Felicitas Rohden’s work “Where the Sun Sets Blue,” we can see these sphere inversions and intermediate surfaces that represent these concurrent exterior and interior spaces.

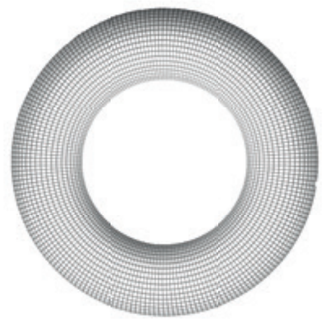
Image Credits:

Fig. 1 Image: Freepik.com
Fig. 2 Mahdavi-Amiri, A., Samavati, F. F. and Peterson, P. “Spherical Geometry of Platonic Solids.“ Wikimedia.org. Wikimedia Commons. Web. 25 February 2015.
Fig. 3 Raymond O. Wells. “Differential and Complex Geometry: Origins, Abstractions and Embeddings.“ Springer Publishing, 2017.
Fig. 4 Crane, Keenan, Ulrich Pinkall, and Peter Schröder. “Robust fairing via conformal curvature flow.“ ACM Transactions on Graphics (TOG) 32.4 (2013): 1-10.
Fig. 5 Lucas Vieira. “Mug and Torus morph.“ Wikimedia.org. Wikimedia Commons. Web. 2 March 2007.
Fig. 6 Novaković, Predrag & Hornak. “3D mesh-triangles with different resolution.” University of Ljubljana, 2017.
Fig. 7 copied from Zuoqin Wang. “Topology of Compact Surfaces.“ Web. 21 June 2021.
Fig. 8 Tttrung. “Klein bottle.” Wikimedia.org. Wikimedia Commons. Web. 18 July 2006.
Fig. 9 The Geometry Center. “Turning a Sphere outside in” 1994. Screenshot by Felicitas Rohden, Retrieved November 5, 2022, from https://youtu.be/OI-To1eUtuU
Fig. 10 The Geometry Center, University of Minnesota. “Outside In Overhead Sequence.” April 1994.
Fig. 11 AugPi. “Morin Surface From The Top.” Wikimedia.org. Wikimedia Commons. Web. 3 July 2005.
Fig. 12 AugPi. “BoysSurfaceTopView.” Wikimedia.org. Wikimedia Commons. Web. 8 November 2004.

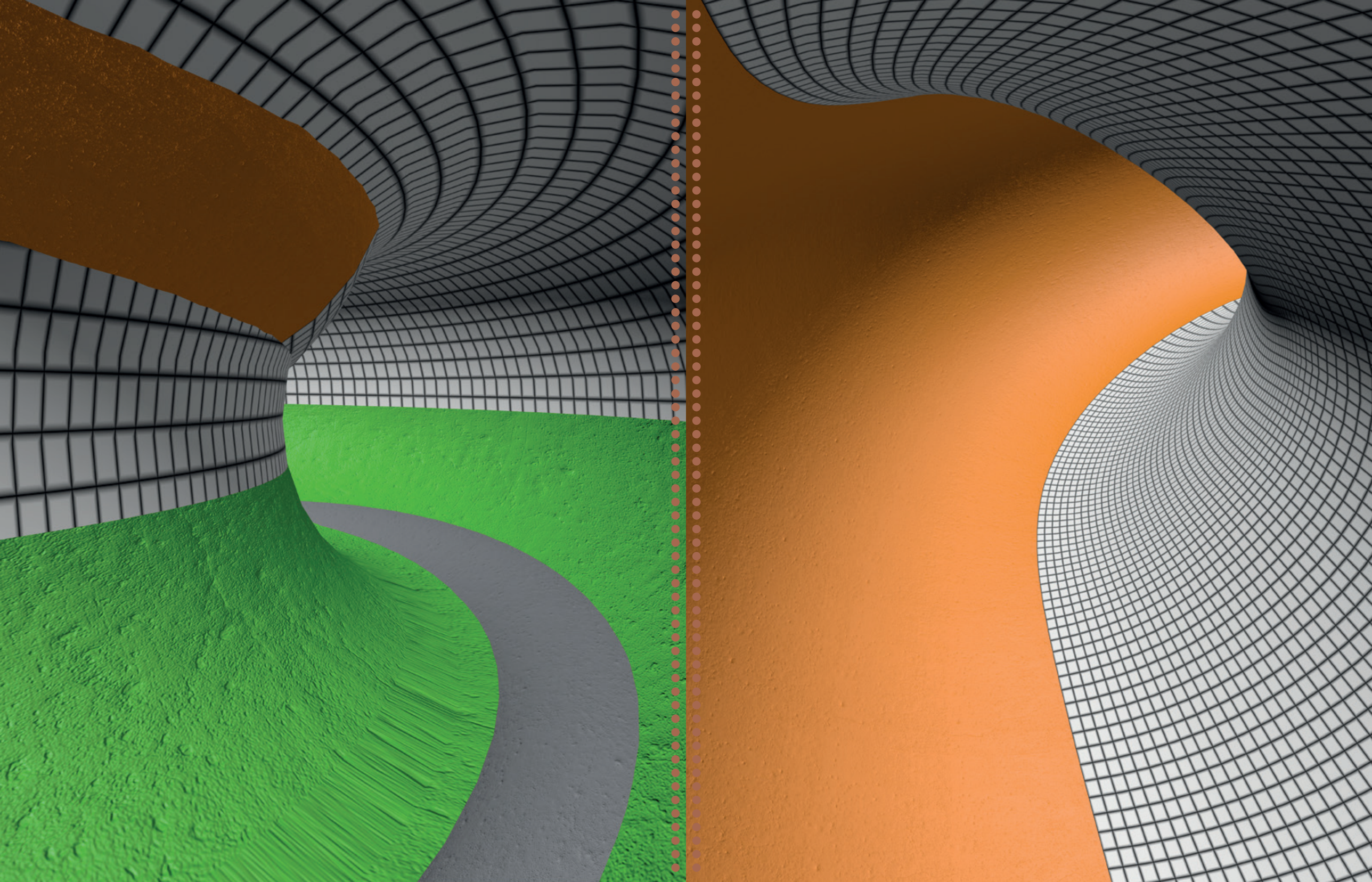


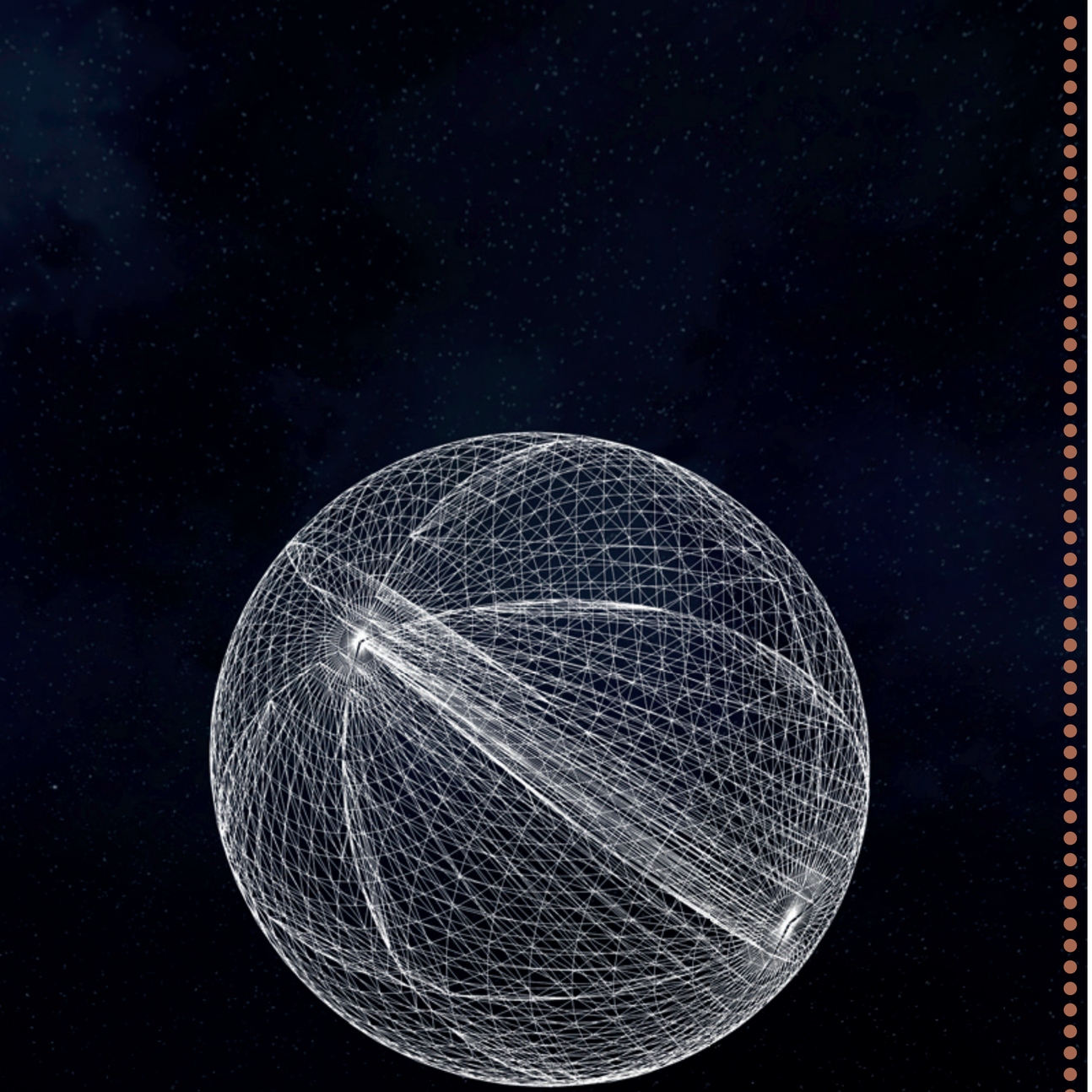






HEARING





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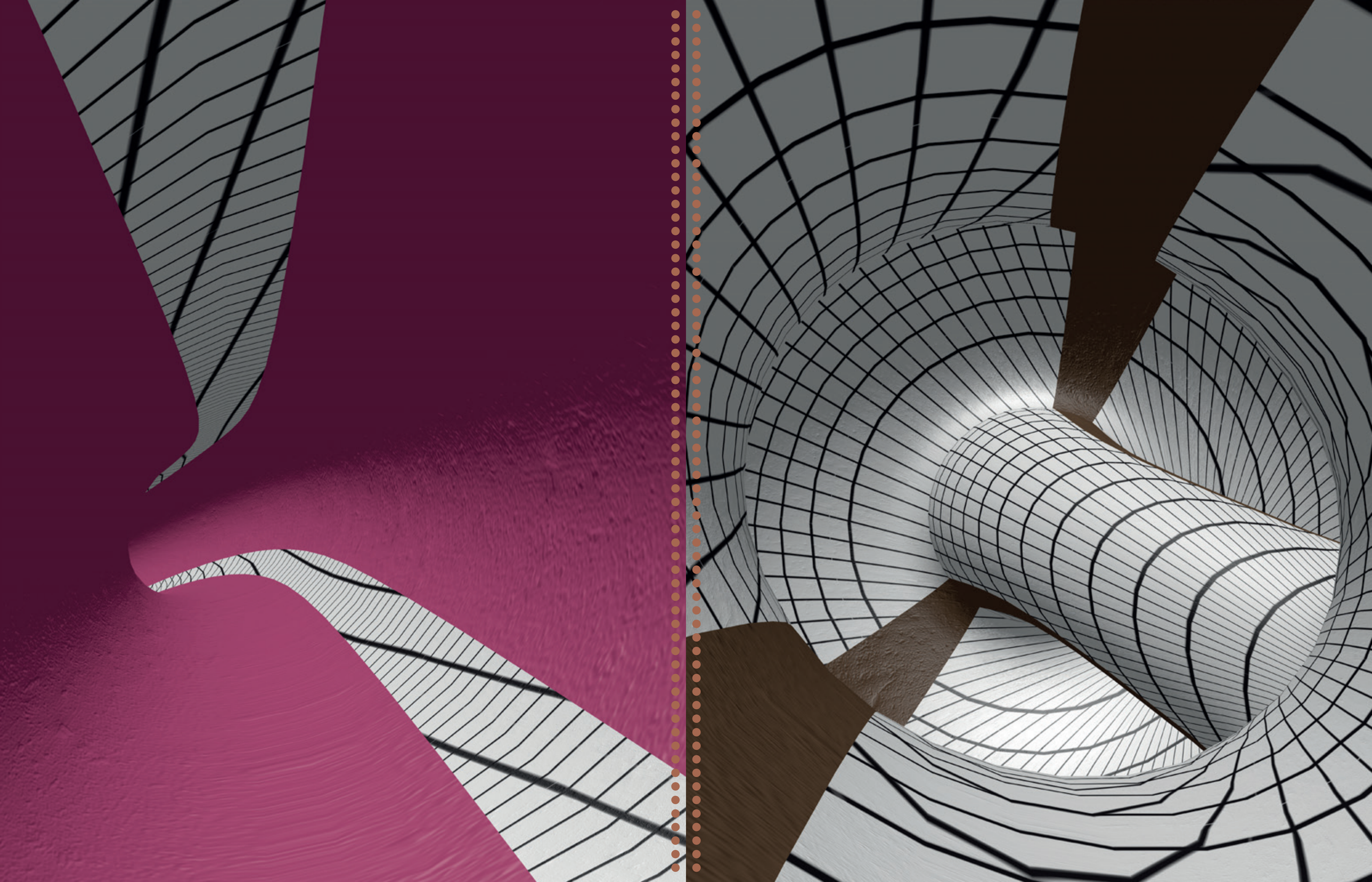

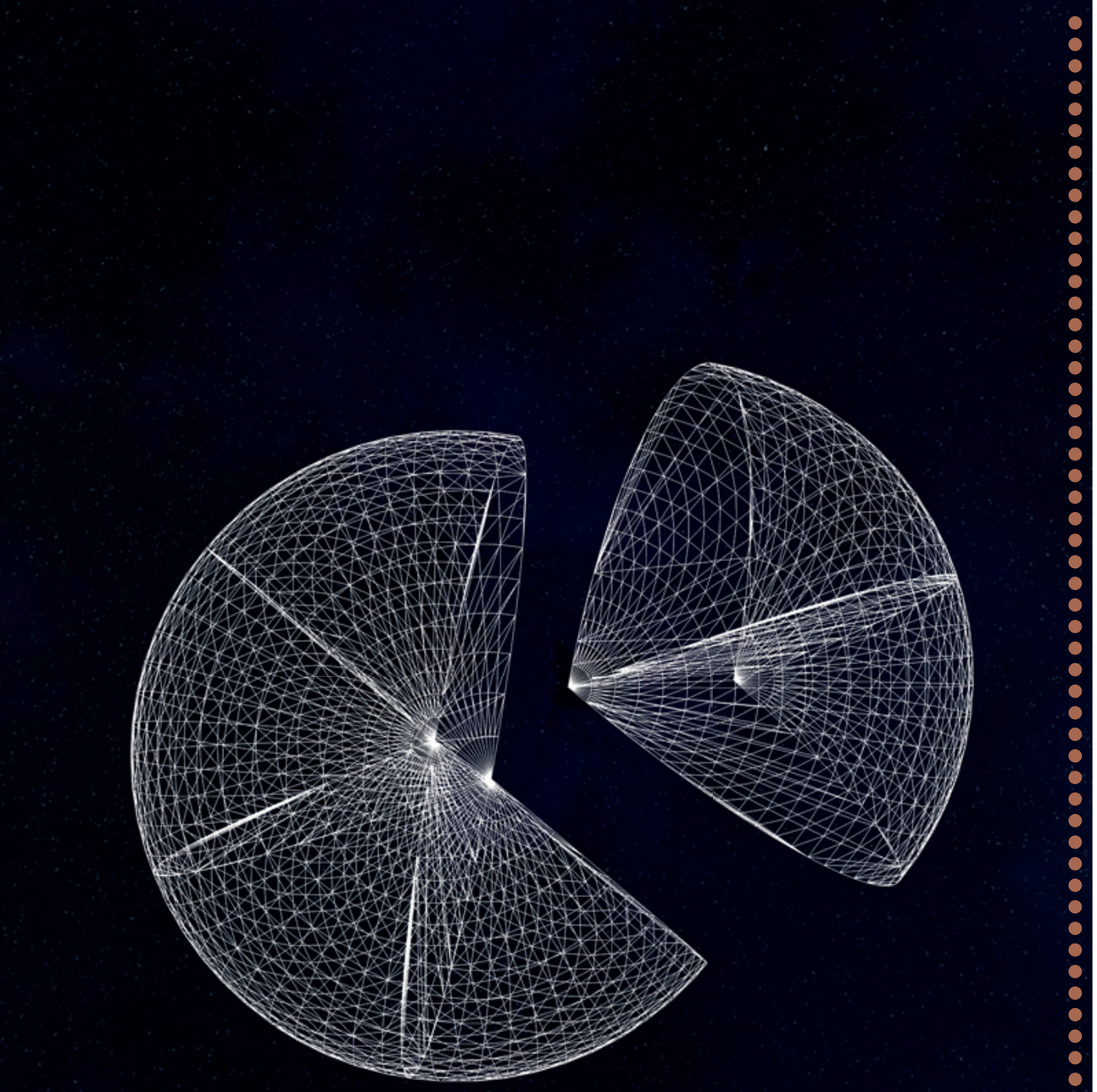

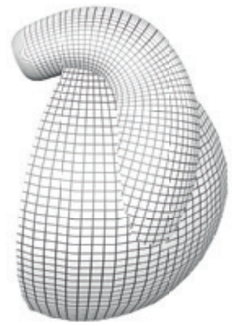

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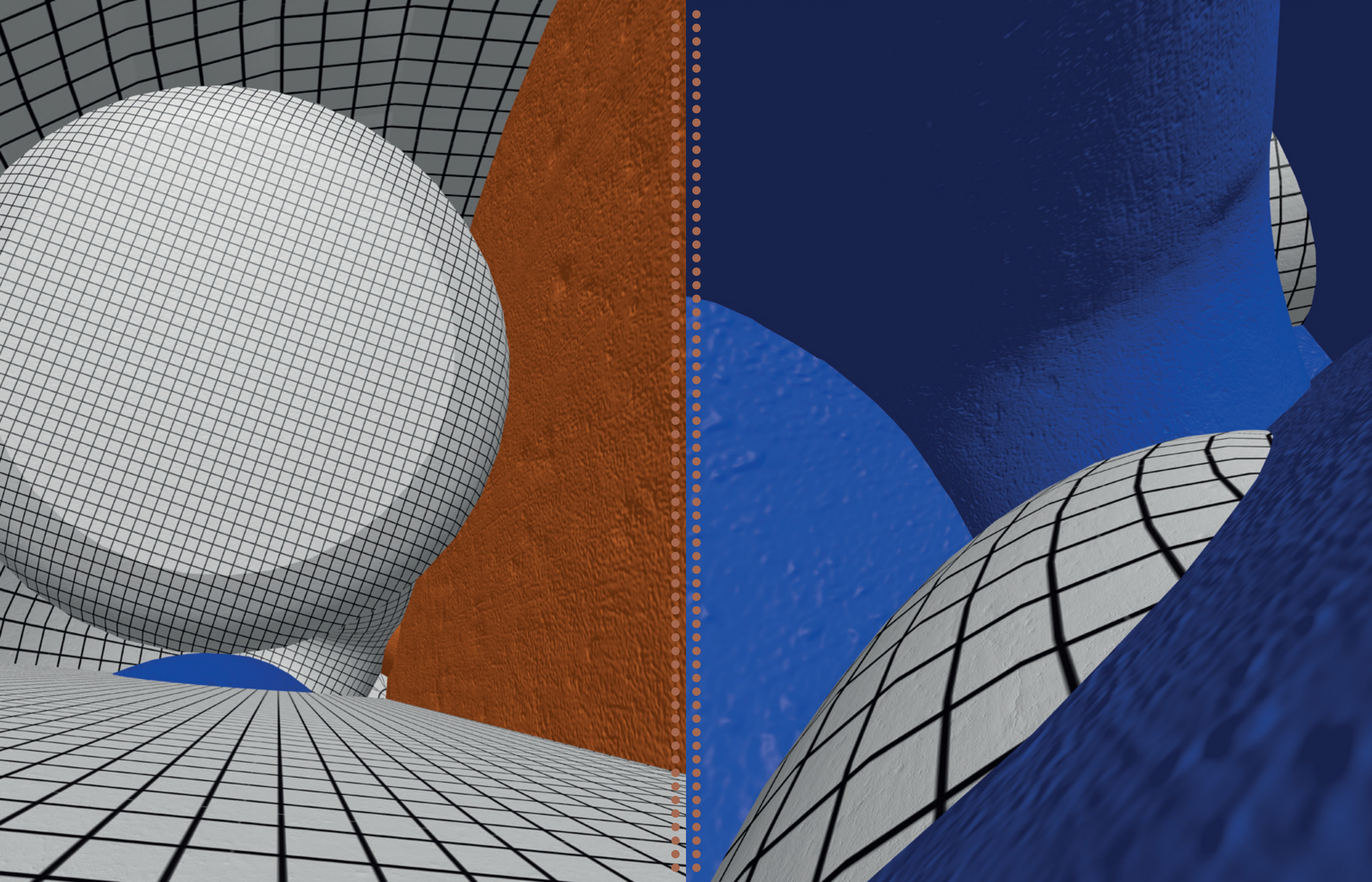

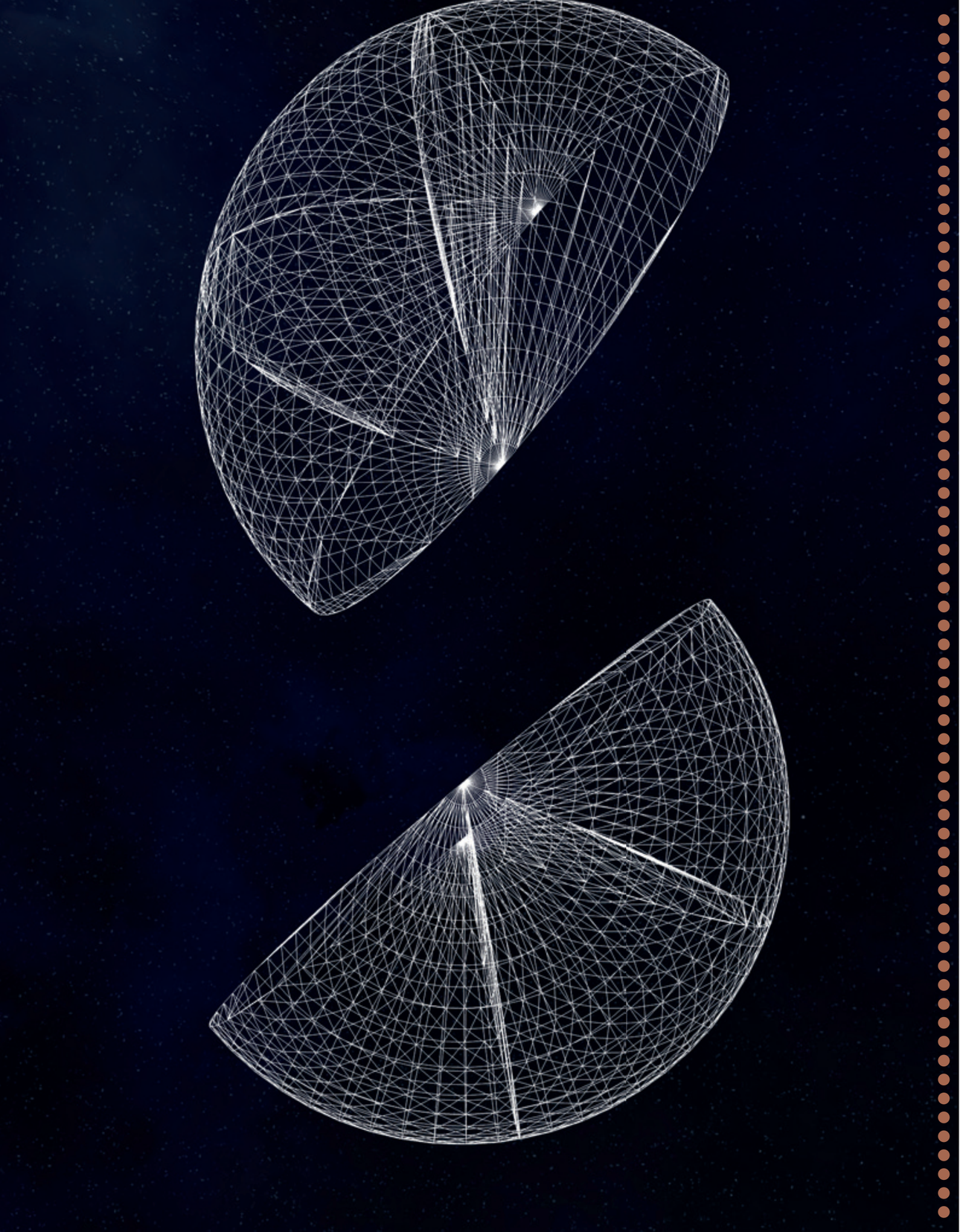

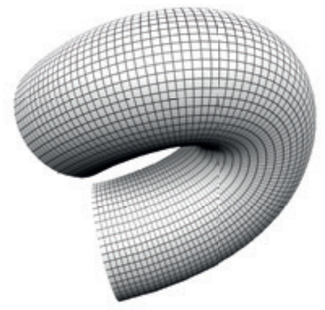

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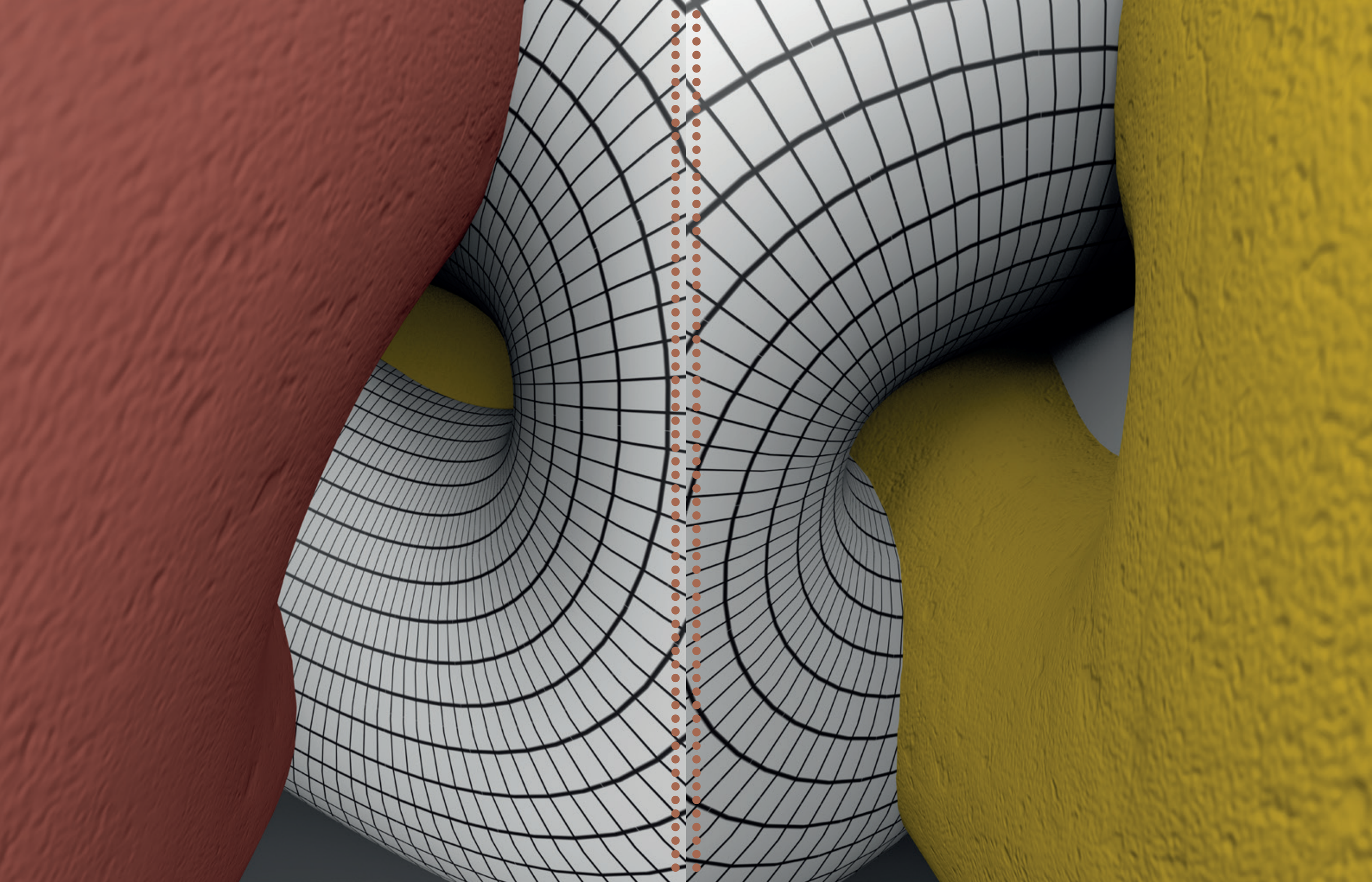

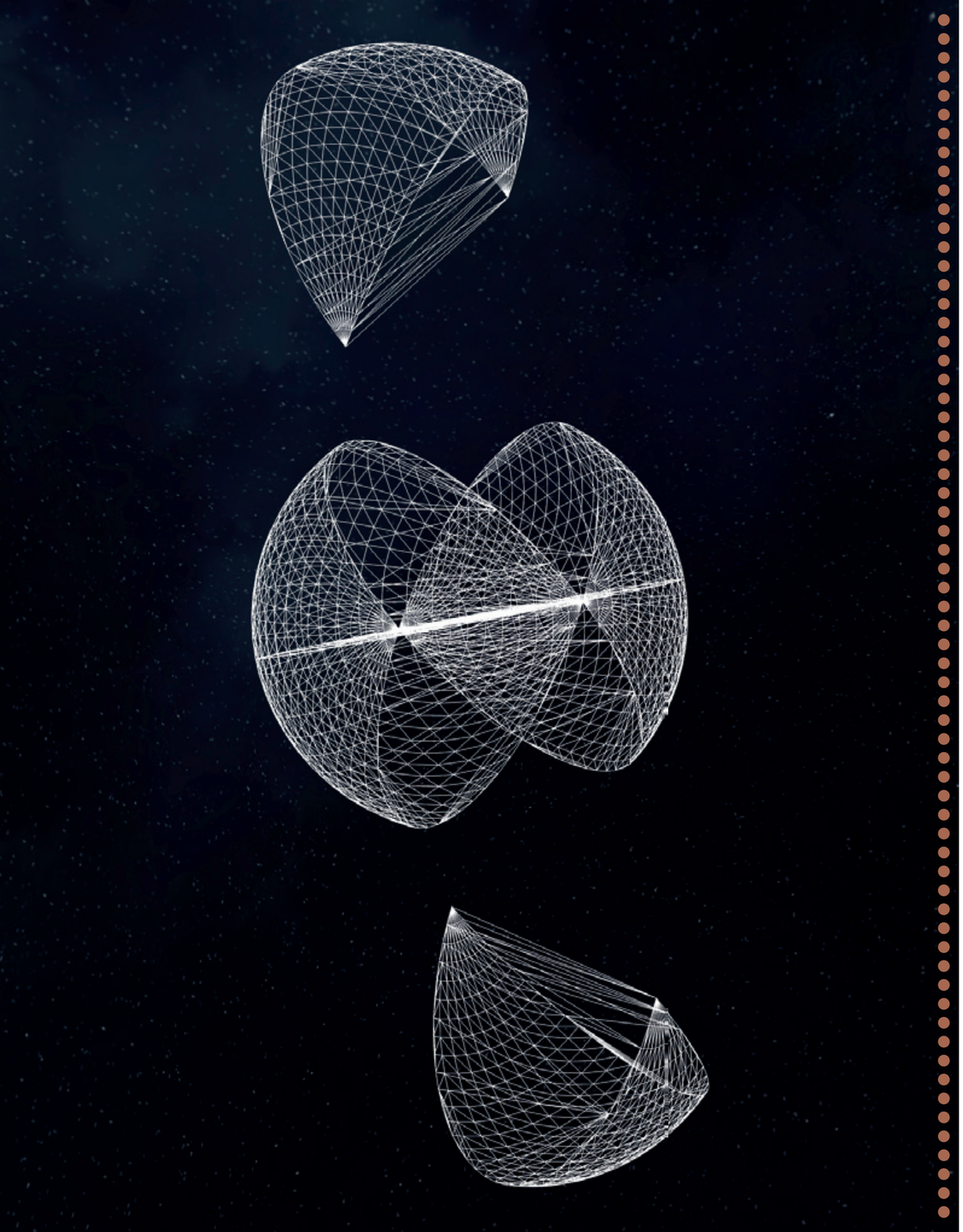

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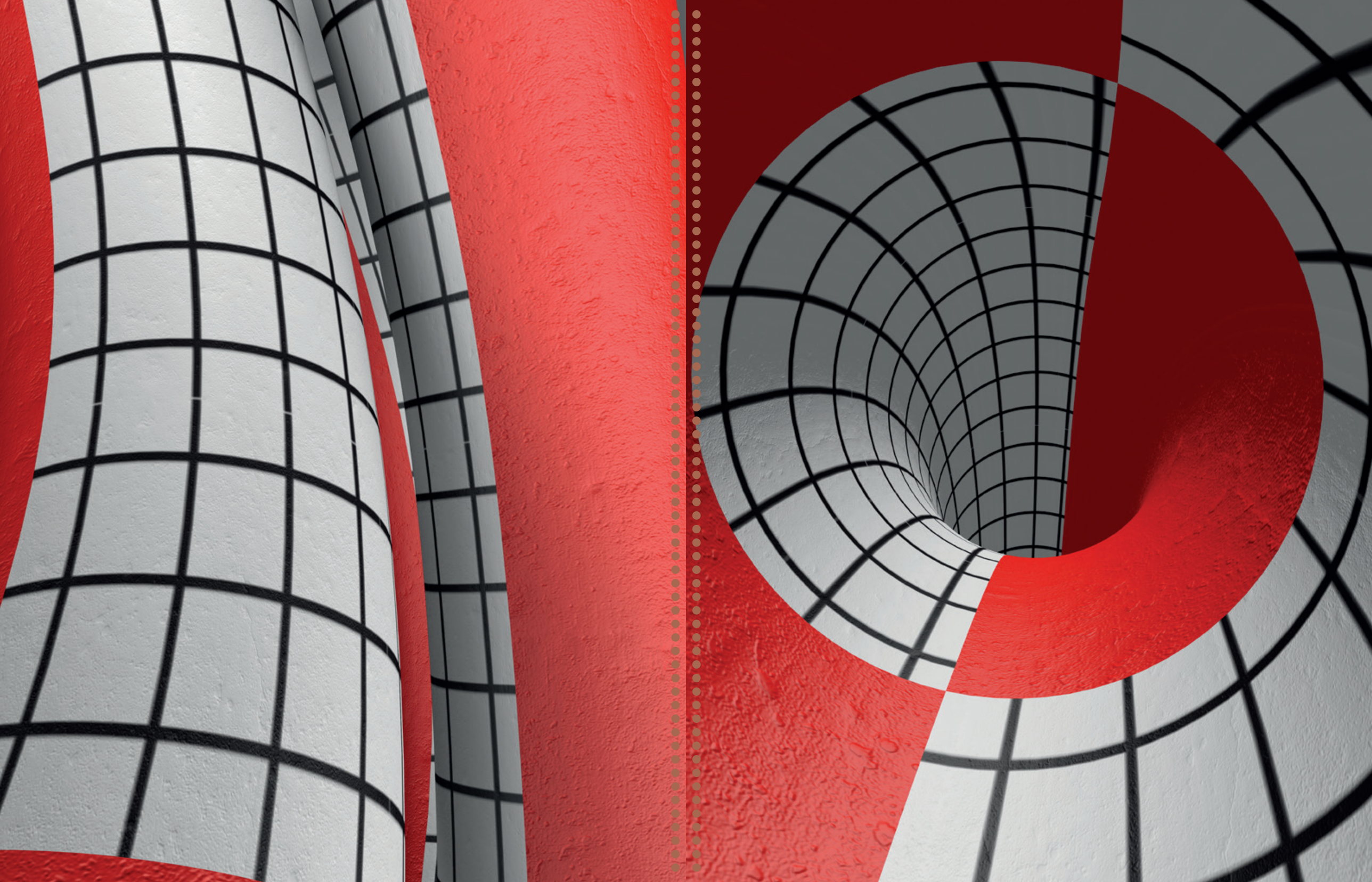

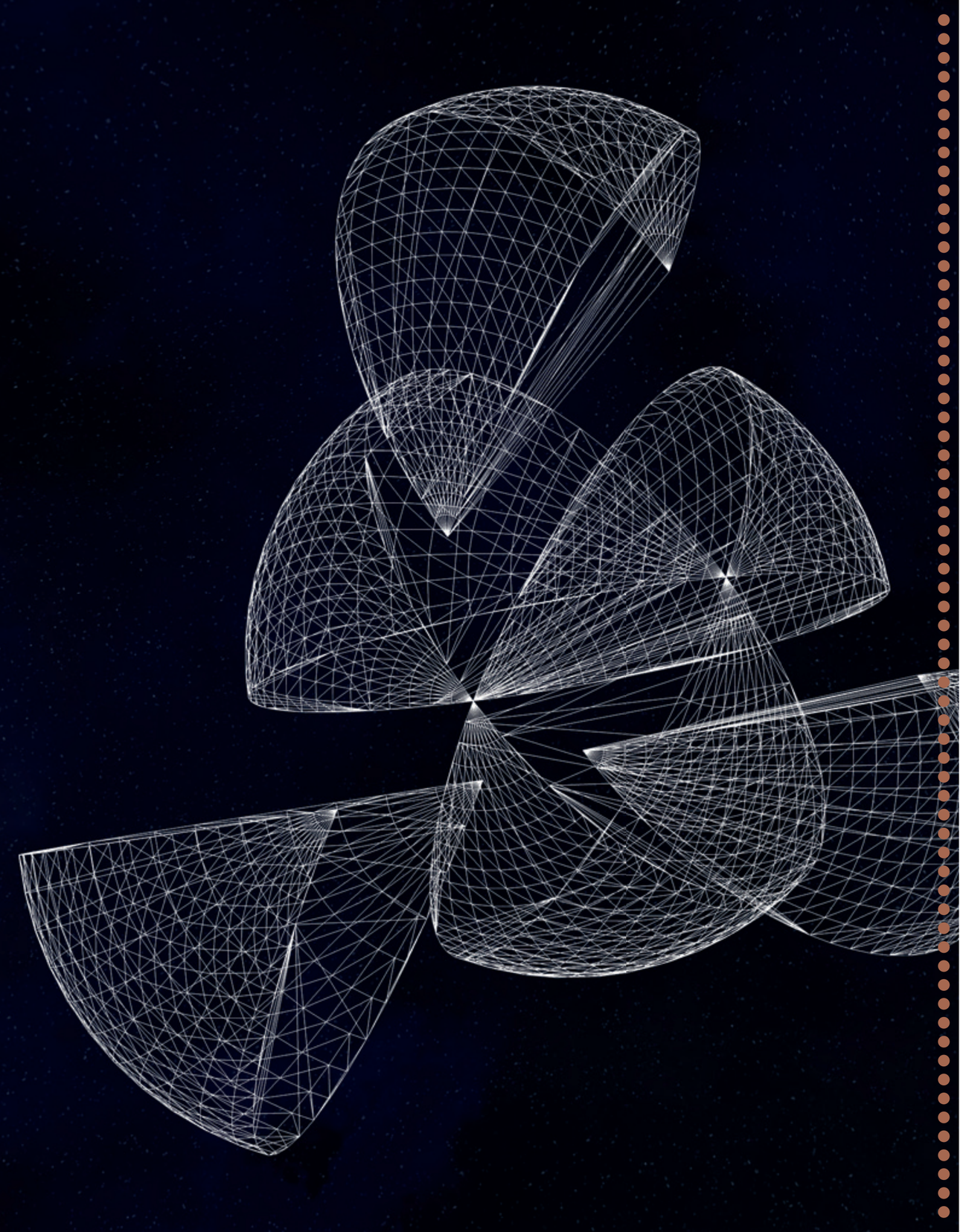

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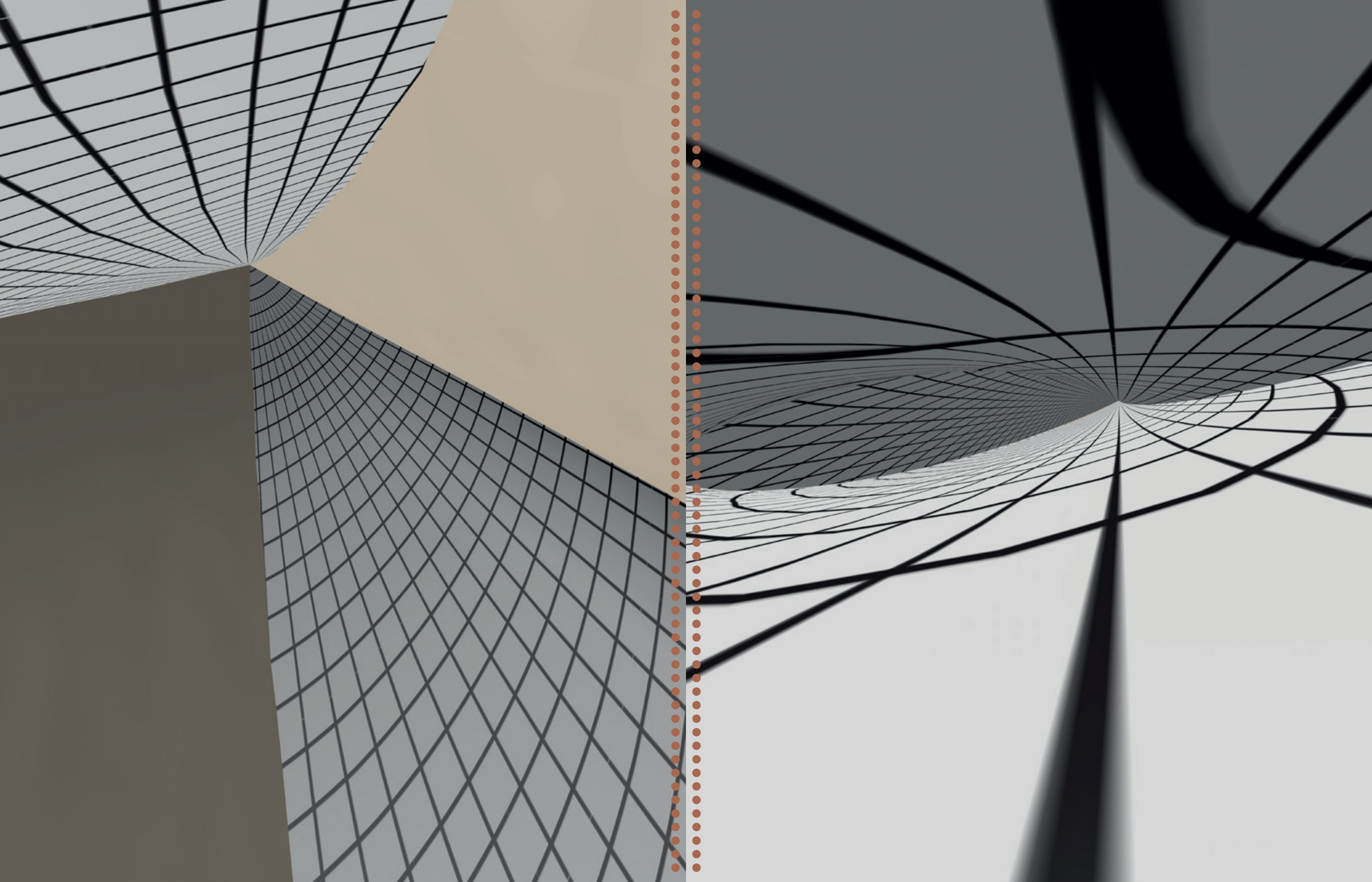

Author Biographies

Felicitas Rohden is a visual artist and deals with the complexity of scientific phenomena and their possible visualisations in her sculptures, artist books and installations.
Her work has been shown internationally in various solo and group exhibitions and she has received several grants and residency opportunities. Next to her artistic practice, she is also a lecturer at the Fine Art Department at Sint Lucas School of Arts in Antwerp. She lives and works in Duesseldorf and Brussels.

Rudi Penne was trained as a mathematician at the University of Antwerp, where he obtained his doctorate in 1992 in a twilight zone between pure mathematics and mechanical applications.
He is a full professor at the University of Antwerp and has regarded science communication as a core task throughout his career. He blogs and writes for Eos, regularly gives lectures and is co-author of the book “De Pracht van Priemgetallen”. He lives and works in Antwerp.

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